Empirical market microstructure research

- measurement of bid-ask spreads
  - stock characteristics
  - location of trading
  - time-of-day or day-of-the-week
- components of the bid-ask spread
  (adverse selection, inventory control,fixed cost)
- price impact of transactions, esp. block trades
  - permanent vs. transitory price effects
- order strategies
- role of inventory in dealership markets

Estimation of the bid-ask spread

Issue: what is the effective bid-ask spread that an average investor pays?

Possible measure: average quoted bid-ask spread. Problems

- quotes may not be binding or for small size only
- spread may vary over the day (U-shape typically)
- bid-ask quote data are not available

Estimation of spread based on transaction prices is usually preferred

Basic microstructure model

Consider the following model of transaction prices

$$
\begin{aligned}
p_t &= m_t + (S/2)Q_t \\
m_t &= m_{t-1} + e_t
\end{aligned}
$$

- $p_t =$ transaction price
- $m_t =$ midpoint of bid and ask quotes
- $Q_t =$ trade direction indicator $\begin{matrix} +1 \text{ if trade is buyer initiated} \\ -1 \text{ if trade is seller initiated} \end{matrix}$
- $e_t =$ new public information arriving between trades
- $S =$ average bid-ask spread

Reduced form of basic model

Typically, m_t is not observed. Taking first differences

$$p_t - p_{t-1} = (S/2)\left(Q_t - Q_{t-1}\right) + e_t \iff \Delta p_t = (S/2)\Delta Q_t + e_t$$

To estimate the spread, run a regression of Δp_t on ΔQ_t

$$\Delta p_t = \beta_0 + \beta_1 \Delta Q_t + e_t$$

The slope coefficient estimates the half-spread $S/2$

To do this we need price and "direction" data on a sequence of trades $t = 1, .., T$

Roll's estimator

If trade direction Q_t is not observed: assuming that Q_t and Q_{t-1} are uncorrelated, Roll shows that

$$cov\left(\Delta p_t, \Delta p_{t-1}\right) = -(S/2)^2$$

Re-arranging this equality gives **Roll's estimator** of the effective bid-ask spread

$$\hat{S} = 2\sqrt{-cov\left(\Delta p_t, \Delta p_{t-1}\right)}$$

Notice that for this estimator to exist, the covariance of price changes must be negative!

Implicit assumption in Roll's estimator: trading does not affect mid-point of bid-ask quote. But there are expectations or inventory effects!

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Incorporating price effects of trading

Basic microstructure model with information effect, based on Glosten-Milgrom model

$$\begin{aligned} p_t &= m_t + (S/2)Q_t \\ m_t &= p_t^* \\ \mu_t &= p_t^* + (1-\pi)(S/2)Q_t \\ p_{t+1}^* &= \mu_t + e_{t+1} \end{aligned}$$

- p_t^* = expected value of the security just before the trade
- μ_t = expected value of the security, given Q_t
- $1-\pi$ = the adverse selection component of the spread

Taking first differences and substituting out m_t and p_t^* gives the reduced form

$$\Delta p_t = (S/2)Q_t - \pi(S/2)Q_{t-1} + e_t$$

The parameters S and π can be estimated from the regression coefficients in a regression of Δp_t on Q_t and Q_{t-1}

An alternative way to write the regression is

$$\Delta p_t = (1-\pi)(S/2)Q_t + \pi(S/2)\Delta Q_t + e_t$$

level variable Q_t estimates adverse selection component: permanent price effect

difference variable ΔQ_t estimates the fixed cost component: temporary price effect

If there is no adverse selection, $\pi = 1$ and the model reduces to the basic microstructure model $\Delta p_t = (S/2)\Delta Q_t + e_t$

Glosten and Harris (1988)

Spread has two components, permanent and transitory

$$\begin{aligned} p_t &= m_t + (C+Z)Q_t \\ \mu_t &= p_t^* + ZQ_t \end{aligned}$$

in the previous notation $C = \pi(S/2)$ and $Z = (1-\pi)(S/2)$

Both spread components are linear in trade size $|q|$

- temporary spread component $C = C_0 + C_1|q|$
- permanent spread component (price impact) $Z = Z_0 + Z_1|q|$

Reduced form

$$\Delta p_t = Z_0 Q_t + Z_1 q_t + C_0 \Delta Q_t + C_1 \Delta q_t + e_t$$

with q_t the "signed" trade size

Data: 800 transactions for 20 NYSE stocks (first 20 in ticker symbol order) starting Dec.1 1981 exclude opening trades

Estimates with restrictions $C_1 = 0$ and $Z_0 = 0$

- Fixed spread component: 4.44 cent per share per trade
- Adverse selection cost: 1.13 cent per share for a 1000 share trade

Estimated spread

- $q = 1000$ shares: 2(4.44+1.13)=11 cents per share of (typically) 20$, around 0.5%, of which adverse selection component: 1.13/5.50 = 20%
- $q = 10,000$ shares: 2(4.44+11.30)=31 cents per share of 20$, around 1.5% Adverse selection component: 11.30/15.50 = 70%

Empirical Inventory Control model

Ho and Macris (1984) propose the empirical model

$$\begin{aligned} p_t &= m_t + (S/2)Q_t \\ m_t &= p_t^* - \beta I_t \\ p_t^* &= p_{t-1}^* + e_t \end{aligned}$$

where I_t is the current inventory level of the dealer.

Reduced form model

$$\Delta p_t = \beta q_{t-1} + (S/2)\Delta Q_t + e_t$$

because the change in inventory $-\Delta I_t$ equals the trade size q_{t-1}

Price effect of trading in same direction as in the Glosten-Milgrom model

Madhavan and Smidt (1991)

Combination of inventory and information based model

$$\begin{aligned} p_t &= m_t + (S/2)Q_t + \lambda q_t \\ m_t &= p_t^* - \beta I_t \\ \mu_t &= p_t^* + (1-\pi)\left[(S/2)Q_t - \beta I_t\right] + \lambda q_t \\ p_{t+1}^* &= \mu_t + e_{t+1} \end{aligned}$$

Differences with Glosten-Harris model

- separate effect of trade size q_t on spread
- inventory affects midpoint of quotes
- inventory level interacts with expectations update

Empirical work Madhavan and Smidt

Estimation of reduced form

$$\Delta p_t = \lambda q_t + (S/2)Q_t - \pi(S/2)Q_{t-1} - \beta I_t + \pi\beta I_{t-1} + e_t$$

Data

- all transactions in 16 stocks of a NYSE specialist
- record of time, price and volume and buy-sell indicator
- inventory constructed by aggregating orders over one month.
  Sample period: 1 Feb 87-31 Dec 87

Empirical results

- inventory effect is rather weak: $\beta = 0$ approximately
- fixed spread component is significant and of similar magnitude for all stocks: $\pi(S/2) = 0.10$
- adverse selection component of fixed part $(1 - \pi)$ is around 0.24
- liquidity parameter λ (for $q = 1000$) is between 0.004 and 0.03
- market is less liquid for small firms