Information Asymmetry, Price Momentum, and the Disposition Effect

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Abstract

Economists have long been puzzled by the tendency of investors to sell winning investments too soon and hold losing investments too long. Several behavioral explanations for this phenomenon, known as the disposition effect, have been advanced. This paper demonstrates that disposition effects are not intrinsically at odds with rational behavior. Specifically, we show (i) that disposition effects arise quite naturally in a world with changing information asymmetry, (ii) that existing empirical tests rejecting an information-based explanation are inconclusive, and (iii) that disposition effects are consistent with price momentum. Further, we derive new empirical implications relating disposition behavior to trading volume, return variability, and price dynamics.

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1 Introduction

Recent empirical studies have documented a number of regularities in the behavior of investors that seem to be at odds with the rational expectations paradigm. One of the most striking patterns is the tendency of investors to sell their winners and to hold on to their losers. Such behavior, which has been termed the “disposition effect” by Shefrin and Statman (1985), has been found in a variety of data sets and time periods.[1]

Although the existence of the disposition effect seems undisputed, economists and investment professionals have not agreed on an explanation for this phenomenon. The empirical literature favors a behavioral explanation offered by Shefrin and Statman (1985), which combines the ideas of mental accounting (Thaler (1985)) and prospect theory (Kahneman and Tversky (1979)). Shefrin and Statman argue that investors keep a separate mental account for each stock. Within that account, investors maximize an “S”-shaped valuation function, which is similar to a standard utility function except that it is defined on gains and losses relative to a reference point (usually the purchase price), rather than on absolute wealth. This valuation function is concave in the gains region and convex in the loss region. Thus, if a stock appreciates in price, the investor’s wealth will be in a more risk-averse part of her valuation function, making a sale more likely. In contrast, if the stock is trading below its purchase price, the investor becomes risk-loving, and will hold on to the stock for a chance to break even.

In addition, there are rational explanations for the disposition effect. First, portfolio rebalancing considerations suggest that investors who do not hold the market portfolio should respond to large price increases by selling some of the shares they hold in these stocks to restore diversification (Lakonishok and Smidt (1986)). Second, since transaction costs tend to be higher for lower priced stocks, and since losing investments are more likely to be lower priced, investors may refrain from selling losing investments simply to avoid

[1]While Shefrin and Statman (1985) restrict their analysis to aggregate data on mutual fund redemptions, recent studies provide direct evidence from actual trading records of individual investors, such as discount brokerage clients (Odean (1998), Ranguelova (2001), Dhar and Zhu (2002)) and professional futures traders (Locke and Mann (2000)). The disposition effect is not particular to the U.S. market. It has also been documented for Finnish investors (Grinblatt and Keloharju (2001)), Israeli investors (Shapira and Venezia (2001)), and investors on the Australian Stock Exchange (Brown et al. (2002)). Weber and Camerer (1998) provide evidence for disposition behavior in an experimental setting.

the higher transaction costs (Harris (1988)). Finally, disposition behavior may result from informational differences across investors (Lakonishok and Smidt (1986)). An investor who purchased a stock on favorable information may sell it when the price goes up because she rationally believes that the stock price now reflects this information. On the other hand, if the price goes down, the investor may continue to hold it, rationally believing that her information has not yet been incorporated into the price.

These alternative rational explanations have been challenged by recent empirical studies. Odean (1998) shows that investors who sell their entire holdings of a stock — and who are thus unlikely to be motivated by diversification — continue to prefer selling winners. In addition, he provides evidence against the hypothesis that higher trading costs for lower priced stocks are responsible for the disposition effect. Even when differences in transaction costs are controlled for, investors appear to be reluctant to realize their losses. Moreover, Odean (1998) and Brown et al. (2002) argue that the investors' preference for realizing winners rather than losers does not appear to be justified by the subsequent stock performance. Both studies find that, on average, winners that are sold outperform, over the subsequent six to 24 months, losers that are not sold, which leads them to reject the information-based explanation suggested by Lakonishok and Smidt.[2]

This paper suggests that the case for rational disposition effects is not hopeless, however. In fact, a simple rational expectations model with asymmetrically informed agents can generate most of the empirically documented facts. The aim of this paper is threefold. First, we demonstrate that, depending on how the degree of information asymmetry changes over time, not only better informed investors choose to sell their winners rather than their losers (as suggested by Lakonishok and Smidt), but also less informed traders might prefer such a strategy. Second, we show that risk-averse investors can rationally exhibit disposition effects even though past winners continue to outperform past losers in subsequent periods. In other words, our results indicate that the existing empirical tests rejecting an information-based explanation for the disposition effect are inconclusive. Third, we examine the relationship between the investors' trading behavior and equilibrium price dynamics. In particular, we show that both a disposition effect and price

[2]While this result can certainly not be considered as evidence in favor of an information-based explanation, we will show in Section 3 that it is not necessarily inconsistent with rational behavior when investors are risk averse.

momentum can arise in a world with fully rational agents.

The basic intuition for our results is as follows. Consider a dynamic model where in each period a fraction of investors receives private information concerning, say, a future earnings announcement or an annual report, and where liquidity shocks prevent prices from fully revealing this information. In such a framework, the informational advantage of privately informed agents can either increase or decrease over time, depending on the precision of their signals and the magnitude of liquidity shocks in each period. If the information disparity decreases over time (either because liquidity shocks become smaller, or because informed agents trade more aggressively and thus reveal more of their private information), less informed investors update their beliefs and hence their asset demands more significantly than better informed investors, as new information becomes available. Market clearing then implies that the latter follow a contrarian strategy: they tend to decrease their stock holdings when good news drives prices up ("sell their winners") and to increase their stock holdings when bad news forces prices down ("hold on to their losers"). The opposite effect is observed when the information asymmetry between investors increases over time. In this case, better informed traders respond more aggressively to new information, forcing less informed traders to pursue a contrarian investment strategy. In other words, less informed investors "underreact" to new information, from the perspective of better informed investors, because the information conveyed by prices is very noisy.

The economic setting we use extends the rational expectations model developed by Grossman and Stiglitz (1980) to two trading periods. However, unlike other dynamic versions of that model in which information is dispersed among many market participants,[3] we maintain the strictly hierarchical information structure of Grossman and Stiglitz's original framework to highlight our ideas in as simple a setting as possible. This admittedly restrictive assumption allows us to characterize equilibrium demands and prices in closed form and to derive conditions for the disposition behavior of various investor groups. Holden and Subrahmanyam (2002) use a similar framework to show that lags in information acquisition can generate positive serial correlation in asset returns.

The remainder of this paper is organized as follows. Section 2 presents

[3]See, e.g., Grundy and McNichols (1989), Brown and Jennings (1989), and He and Wang (1995) for a dynamic setting with a single risky asset, and Brennan and Cao (1997) and Zhou (1998) for the multiple-asset case.

the economic setting. Section 3 describes the equilibrium of the model and derives prices and demands in closed form. The necessary conditions for investors to exhibit disposition effects are provided in Section 4. Section 5 analyzes the relationship between the investors' trading behavior and asset price dynamics. Section 6 concludes the paper. All proofs are contained in the Appendix.

2 The Model

We consider a two-period exchange economy. Agents trade at dates 0 and 1 and consume at date 2. In each period, a fraction of investors receives private information concerning the assets' future payoffs. There is some noise in the market, in the form of supply shocks, that prevents equilibrium prices from fully revealing the investors' private information. The structure of the economy is common knowledge.

2.1 Assets

There are two assets available for trading in the market: a riskless bond and a risky stock. The bond is in perfectly elastic supply. For simplicity, we normalize its interest rate to zero.

Each share of the stock pays a liquidation value of V at date 2, which is drawn from a normal distribution with mean zero and variance one, and is unknown to investors prior to date 2. Shares of the stock are infinitely divisible and are traded competitively in the stock market. The price of the stock at date $t = 0, 1$ is denoted by P_t. The aggregate supply of the stock is random and equals z_0 at date 0 and $z_0 + z_1$ at date 1. Such supply shocks are a typical ingredient of rational expectations models. The noise they create prevents equilibrium prices from fully revealing the informed agents' private information. The assumption of a stochastic stock supply is equivalent to assuming the presence of liquidity traders who have inelastic demands of $-z_0$ and $-z_1$ shares of the stock, for reasons that are exogenous to the model. For simplicity, we assume that z_0 and z_1 are normally distributed with mean zero[4] and variances $\sigma^2_{z_0}$ and $\sigma^2_{z_1}$, respectively, and are independent of each

[4]Normalizing the expected stock supply to zero can be done without loss of generality. Introducing a positive mean supply would cause the unconditional risk premium to be non-zero, but would not alter our basic results.

other and all other random variables.[5]

The assumption that all uncertainty is resolved by date 2 is not crucial for our results. In a more realistic setting, the payment of the liquidating dividend could be interpreted as any public announcement (such as an earnings announcement) that temporarily eliminates, or at least reduces, informational differences among investors. As long as information is made public in the form of discrete news events, rather than continuously, the introduction of an infinite series of public announcements would not alter our basic conclusions.[6]

2.2 Investors

Our economy is populated by two types of market participants: informed investors who possess private information about the stock's payoff, and uninformed investors.

In each period, informed investors receive a private signal that is related to the stock's payoff as follows:

$$S_t = V + \epsilon_t, \qquad t = 0, 1$$

where the error terms $\{\epsilon_t\}_{t=1,2}$ are independently normally distributed with mean zero and variance $\sigma^2_{\epsilon_t}$, conditional on V. Thus, the investors' signals are unbiased forecasts of V. Uninformed investors do not observe S_0 and S_1 directly, but can infer some of the informed agents' private information from market prices.

For tractability, we assume that informed and uninformed investors behave competitively. They take equilibrium prices as given even though their aggregate trades affect market prices. Such behavior can be justified by assuming that investors are individually infinitesimal, so that no single trader can influence the price. More precisely, we assume that there is a continuum of informed (uninformed) investors whose set has measure M (N). Since M and N can be interpreted as the proportions of each investor type, we normalize $M + N = 1$. It is also common knowledge that the endowment of every trader is equal to zero.

Following Grossman and Stiglitz (1980), we assume that both informed and uninformed investors have negative exponential utility over terminal

[5]This is equivalent to assuming that the demand of liquidity traders is independent over time.

[6]See Holden and Subrahmanyam (2002) for a formalization of this approach.

wealth, with a common risk aversion coefficient γ, that is, $U(W) = -e^{-\gamma W}$. As well as allowing us to derive linear equilibria in closed form, this utility function has the advantage that income effects play no role in the agents' portfolio choice.[7] This ensures that the disposition effect is not driven by the agents' need to rebalance their portfolios as a result of price changes.[8]

3 Equilibrium

In this section, we solve for the equilibrium of the economy defined above. The equilibrium concept we use is that of a rational expectations equilibrium (REE), developed by Grossman (1976), Hellwig (1980), and Bray (1981). Formally, an REE is defined by prices P_0 and P_1, and by demand functions of informed and uninformed investors, such that: (i) for each price-taking investor, the trades specified by her demand function at a given date maximize her expected utility of consumption, subject to a budget constraint and available information, including past and current market prices; and (ii) for every combination of signals and supply shocks, markets clear.

Given the well-known properties of CARA preferences under normal distributions of payoffs, signals, and supply shocks, we restrict our attention to linear equilibria. Thus, we postulate that the prices are linear functions of the private signals and the supply shocks to date, such that:

$$P_0 = a\,S_0 + b\,z_0, \tag{1}$$

$$P_1 = c\,S_0 + d\,S_1 + e\,z_0 + f\,z_1. \tag{2}$$

In the ensuing analysis, we derive a linear equilibrium in which this conjecture is confirmed to be correct. Although the aggregate stock supply is independent of the stock's payoff, V, it enters the price functions because it affects the number of shares held by investors and, hence, the total risk the economy has to bear.

[7]See, e.g., Huang and Litzenberger (1988).

[8]If, for example, the utility function exhibited constant relative risk aversion, investors would respond to large price increases by selling some of their shares in order to restore the initial proportions of wealth invested in the risky and the riskless assets.

3.1 Optimal demand of informed investors

The optimal demand of informed investors can be derived using backward induction. Each investor's final wealth at date 2 is given by:

$$W_I = x_0(V - P_0) + x_1(V - P_1),$$

where x_t denotes the investor's date t demand. Since W_I is normally distributed, conditional on the informed investor's date 1 information set $\mathcal{F}_1^I = \{S_0, S_1, P_0, P_1, z_0, z_1\}$[9], one can use the mean-variance framework to show that the optimal demand for shares at date 1 is equal to:

$$x_1 = \frac{E\left[V \mid \mathcal{F}_1^I\right] - P_1}{\gamma \, Var\left[V \mid \mathcal{F}_1^I\right]} - x_0 = \frac{\lambda_0 \, S_0 + \lambda_1 \, S_1 - P_1}{\gamma \, \tau_I} - x_0, \qquad (3)$$

where the values of the constants λ_0, λ_1, and τ_I (which are provided in the Appendix) can be calculated from the projection theorem.[10] The right-hand side of this equation shows the familiar result that CARA preferences under normal distributions of payoffs lead to linear optimal demand functions.

When making their portfolio decisions at date 0, informed investors can use their signal S_0 and the observed market price P_0 to predict future returns, i.e., $\mathcal{F}_0^I = \{S_0, P_0, z_0\}$. The following lemma shows that the date 0 demand of informed investors, x_0, is linear in the expected date 1 price change, $E[P_1 - P_0 \mid \mathcal{F}_0^I]$, and in the expected date 2 price change, $E[V - P_1 \mid \mathcal{F}_0^I]$.

Lemma 1 *At date 0, the optimal demand of informed investors is given by:*

$$x_0 = \frac{E\left[P_1 \mid \mathcal{F}_0^I\right] - P_0}{\gamma \, G_{11}^I} + \frac{G_{11}^I - \lambda_1 \, G_{12}^I}{G_{11}^I} \, \frac{E\left[V - P_1 \mid \mathcal{F}_0^I\right]}{\gamma \, \tau_I}, \qquad (4)$$

where G_{11}^I and G_{12}^I are functions of the price coefficients d and f.

Note that the second component of the demand function is proportional to the investor's expected optimal stock holdings at date 1, $E\left[x_0 + x_1 \mid \mathcal{F}_0^I\right]$: on the one hand, informed investors attempt to exploit the expected price appreciation across dates 0 and 1; on the other hand, they hedge in advance

[9]The information set contains the current price P_1, since investors can submit their demands as a function of the price. Moreover, knowing S_0 and S_1, informed investors can infer the supply shocks z_0 and z_1 from equilibrium prices.

[10]See, e.g., Anderson (1984), chapter 2.

the anticipated demand at date 1. The constant $(G^I_{11} - \lambda_1 G^I_{12})/G^I_{11}$ represents the degree to which informed traders hedge their expected date 1 demand in advance. In fact, it is easy to show that this coefficient lies between zero and one, and increases (decreases) with the precision of the date 0 (date 1) signal S_0 (S_1). The coefficient equals one if either S_0 fully reveals V (i.e., if $\sigma_{\epsilon_0} = 0$) or if S_1 contains no additional information (i.e., if $\sigma_{\epsilon_1} = \infty$).

3.2 Optimal demand of uninformed investors

At date 1, an uninformed investor faces the following optimization problem:

$$\max_{y_1} E\left[-\exp\left\{-\gamma\left(y_0(V - P_0) + y_1(V - P_1)\right)\right\} \mid \mathcal{F}^U_1\right],$$

where y_t denotes her date t demand. Since uninformed investors cannot observe the signals S_0 and S_1, their only source of information about the payoff V is past and current market prices, i.e., $\mathcal{F}^U_1 = \{P_0, P_1\}$. We show in Section 3.3 that technical analysis does have value in our economy, that is, the past price P_0 does affect investors' date 1 beliefs (except for a set of parameter values with measure zero).

Given the linearity of the equilibrium pricing relations, the random variables V, P_0, and P_1 are jointly normally distributed, which implies that the investor's terminal wealth conditional on $\mathcal{F}^U_1$ is normally distributed as well. Therefore, one can use the standard mean-variance analysis to show that the uninformed investor's optimal date 1 demand is given by:

$$y_1 = \frac{E\left[V \mid \mathcal{F}^U_1\right] - P_1}{\gamma\, Var\left[V \mid \mathcal{F}^U_1\right]} - y_0 = \frac{\kappa_0\, P_0 + (\kappa_1 - 1)\, P_1}{\gamma\, \tau_U} - y_0. \tag{5}$$

The conditional moments of V can again be calculated from the projection theorem (see equations (11) and (12) in the Appendix).

At date 0, uninformed investors can infer information about V from the observed market price P_0, i.e., $\mathcal{F}^U_0 = \{P_0\}$. Lemma 2 shows that their demand, y_0, is linear in $E[P_1 - P_0 \mid \mathcal{F}^U_0]$ and $E[V - P_1 \mid \mathcal{F}^U_0]$ and, hence, in P_0.

Lemma 2 *At date 0, the optimal demand of uninformed investors is given by:*

$$y_0 = \frac{E\left[P_1 \mid \mathcal{F}^U_0\right] - P_0}{\gamma\, G_U} + \frac{1 - \kappa_1}{G_U}\, \frac{E\left[V - P_1 \mid \mathcal{F}^U_0\right]}{\gamma\, \tau_U}, \tag{6}$$

where G_U is a function of the price coefficients a, b, c, d, e, and f.

Note that the optimal demand of uninformed traders, like that of informed traders, consists of two components: the first component exploits the price change in the first period, and the second hedges in advance the expected demand in the second period.

3.3 Characterization of linear equilibrium

Solving for a rational expectations equilibrium entails determining values for the price coefficients a, b, c, d, e, and f, such that the market-clearing conditions

$$M\,x_t + (1-M)\,y_t = z_t, \quad \text{for } t = 0, 1,$$

are satisfied with probability one. The following theorem presents closed-form solutions for these coefficients.

Theorem 1 *There exists a linear REE, in which the coefficients of the price functions P_0 and P_1 are given by:*

$$\begin{aligned}
a &= \frac{M}{D_0}\,(M + \gamma^2\,\sigma^2_{\epsilon_0}\,\sigma^2_{z_0}),\\
b &= -\frac{\gamma\,\sigma^2_{\epsilon_0}}{M}\,a,\\
c &= \frac{M\,\sigma^2_{\epsilon_1}}{D_1}\,(M + \gamma^2\,\sigma^2_{\epsilon_0}\,\sigma^2_{z_0})(M^2 + \gamma^2\,\sigma^2_{\epsilon_1}\,\sigma^2_{z_1}),\\
d &= \frac{M\,\sigma^2_{\epsilon_0}}{D_1}\,(M^2 + \gamma^2\,\sigma^2_{\epsilon_0}\,\sigma^2_{z_0})(M + \gamma^2\,\sigma^2_{\epsilon_1}\,\sigma^2_{z_1}),\\
e &= -\frac{\gamma\,\sigma^2_{\epsilon_0}}{M}\,c,\\
f &= -\frac{\gamma\,\sigma^2_{\epsilon_1}}{M}\,d,
\end{aligned}$$

with

$$\begin{aligned}
D_0 &= M^2(1 + \sigma^2_{\epsilon_0}) + \gamma^2(M + \sigma^2_{\epsilon_0})\,\sigma^2_{\epsilon_0}\,\sigma^2_{z_0},\\
D_1 &= \sigma^2_{\epsilon_0}\sigma^2_{\epsilon_1}\delta_0\,\delta_1 + M(\sigma^2_{\epsilon_0} + \sigma^2_{\epsilon_1})\,\delta_0\,\delta_1 + M^2\big(\sigma^2_{\epsilon_0}(1 + \sigma^2_{\epsilon_1})\,\delta_0 + \sigma^2_{\epsilon_1}(1 + \sigma^2_{\epsilon_0})\,\delta_1\big)\\
&\quad + M^3(\sigma^2_{\epsilon_1}\delta_0 + \sigma^2_{\epsilon_0}\delta_1) + M^4(\sigma^2_{\epsilon_0} + \sigma^2_{\epsilon_1} + \sigma^2_{\epsilon_0}\sigma^2_{\epsilon_1}),
\end{aligned}$$

where $\delta_0 = \gamma^2\,\sigma^2_{\epsilon_0}\,\sigma^2_{z_0}$ and $\delta_1 = \gamma^2\,\sigma^2_{\epsilon_1}\,\sigma^2_{z_1}$.

There may be multiple equilibria in our model, even within the class of linear equilibria. The reason is that calculating the equilibrium price coefficients involves solving a quintic equation, as the proof of Proposition 1 shows. The following proposition identifies sufficient conditions for the equilibrium characterized in Theorem 1 to be the unique linear REE.

Proposition 1 *The equilibrium specified in Theorem 1 is the unique linear REE if* $M \geq \frac{2}{\sqrt{3}} - 1 \, (\approx 0.155)$ *and if*

$$\sigma_{\epsilon_1} \leq \frac{2\sqrt{M}}{1-M} \, \sigma_{\epsilon_0}.$$

4 Trading Behavior

This section describes the trading behavior of informed and uninformed investors and derives empirical implications. We show that, depending on how the degree of information asymmetry between informed and uninformed investors changes over time, both types of investors can exhibit disposition effects.

Before we present our results, we have to clarify what "selling winners" and "holding on to losers" means in the context of our model. Since individual investors typically do not engage in short-selling activities, most empirical studies focus only on long positions when calculating various measures for the disposition effect. There is no need to do so in our model: "selling stocks" simply refers to any situation in which the initial date 0 position is (partially) reversed at date 1. Thus, we will use the term "selling a winner" for situations in which an initial long position is reduced by selling stocks after a price increase, as well as for situations in which an initial short position is reduced by buying back shares after a price decrease. Furthermore, we will use the term "holding on to a stock" to describe all situations where the initial (long or short) position in a stock remains constant[11] or increases in the second trading round.

Most empirical studies measure the disposition effect by calculating the difference between the investors' propensity to realize winners and their propensity to realize losers in their portfolios. Specifically, they compare

[11] While individual investors usually do not rebalance their portfolios on a daily basis, the agents in our model almost certainly change their portfolio holdings at date 1 (i.e., with probability one).

the proportion of gains realized (PGR), i.e., the number of winning stocks sold divided by the total number of winning stocks in a portfolio, with the proportion of losses realized (PLR), i.e., the number of losing stocks sold divided by the total number of losing stocks in a portfolio. A stock is considered a winner (loser), if its price is higher (lower) than its purchase price at the time of calculation.[12] A positive difference between PGR and PLR indicates that investors prefer to realize their winning investments. Note that this measure is not affected by price trends, since it compares the frequency with which investors sell their winning and losing investments relative to their opportunities to sell each type.[13] In our model, the theoretical analog of the PGR measure is the conditional probability that a winning stock is sold at date 1.[14] The PLR measure corresponds to the conditional probability that a losing investment is sold at date 1.

4.1 Trading behavior of informed investors

There are two reasons why we might expect informed investors to exhibit disposition effects. First, selling stocks at high prices (likely winners) and buying stocks at low prices (likely losers) always seems a good idea when some investors trade for non-informational motives. In our model, changes in the aggregate supply are completely independent of the stock's date 2 payoff. Interpreting these supply shocks as the result of liquidity trading activity, we would expect rational risk-averse investors to absorb these shocks by providing liquidity at an appropriate risk premium. Investors are willing to buy (sell) stocks at prices below (above) the expected fundamental value. Since stocks with high (low) date 1 prices are, on average, winners (losers), informed investors are more likely to sell winners than losers if the variance of the date 1 supply shock is high enough.[15]

The second reason is related to differences between the informed and uninformed investors' information about the asset payoff, V. Specifically,

[12] For a more detailed description of the methodology see Odean (1998).

[13] In an upward-moving market, the number of opportunities to realize a gain will far exceed the number of opportunities to realize a loss. In this case, an observation of more realized gains than losses does not necessarily arise, because investors prefer to realize their winning investments.

[14] Formally, for informed investors the event "sell a winning stock" is defined as $\{\omega \in \Omega : (x_0 > 0 \wedge \Delta P_1 > 0 \wedge x_1 < 0) \vee (x_0 < 0 \wedge \Delta P_1 < 0 \wedge x_1 > 0)\}$. The other events are defined analogously.

[15] Since this result is almost trivial, we do not give a formal proof.

informed investors may exhibit disposition effects if their informational advantage over uninformed investors sufficiently decreases. For example, consider the extreme scenario where the signal S_1 is made public at date 1, and suppose that informed traders purchased the stock on the basis of favorable information at date 0. If S_1 is good news as well, both informed and uninformed investors will revise their expectations about the payoff V upwards, and demand higher stock holdings at date 1. This drives the price up. Although both types of investors respond to good news by demanding more stocks, they do so with different intensities. Not knowing S_0, uninformed investors learn more from the signal S_1 than informed investors do. In other words, the announcement of S_1 has a much stronger impact on the uninformed investors' expectations about V and hence on their optimal demand.[16] In equilibrium, informed investors will therefore find it optimal to sell part of their stock holdings to uninformed investors at a price that exceeds their private valuations. To put it differently, from the informed investors' perspective the price increase is not justified by the new information S_1, so they decide to sell shares. If, on the other hand, S_1 signals a low payoff, both informed and uninformed investors want to sell shares. Again, however, uninformed investors, not knowing the favorable signal S_0, are more eager to do so. The price will drop until informed investors are willing to buy shares from uninformed investors despite the bad signal S_1 and the market clears. This implies that, on average, informed investors react to a price increase by selling some of their shares and to a price decrease by buying even more shares.

In general, the information asymmetry decreases from date 0 to date 1, if the variance of the date 1 supply shock z_1 is low relative to that of the date 0 shock z_0, the precision of the signal S_1 is high relative to that of the signal S_0, and the fraction of informed traders, M, is high. The following proposition shows that even in the absence of a date 1 supply shock, informed traders can show disposition effects.

Proposition 2 *Suppose*

$$\sigma_{\epsilon_1}\sigma_{z_1} < \frac{M\,\sigma_{\epsilon_0}\sigma_{z_0}}{\sqrt{M^2\left(1+\sigma_{\epsilon_0}^2\right)+\gamma^2\sigma_{\epsilon_0}^4\sigma_{z_0}^2}} \equiv \hat{\sigma}_1.$$

[16] It is straightforward to show that $Var[V \mid \mathcal{F}_0^U] - Var[V \mid \mathcal{F}_0^I] > Var[V \mid \mathcal{F}_1^U] - Var[V \mid \mathcal{F}_1^I]$ if there is no date 1 supply shock.

Then, informed investors are more likely to sell their winning stocks than their losing stocks, i.e.,

$$Pr\big(x_1 < 0 \mid x_0 > 0, \Delta P_1 > 0\big) > Pr\big(x_1 < 0 \mid x_0 > 0, \Delta P_1 < 0\big),$$

where ΔP_1 denotes the date 1 price change $P_1 - P_0$.

Note that, since the random variables x_0, x_1, and ΔP_1 are jointly normally distributed with mean zero, it follows immediately that Proposition 2 applies to short sales as well. If informed investors initially have a short position in the stock, they are more likely to buy back shares when the price goes down than when the price goes up, i.e.,

$$Pr\big(x_1 > 0 \mid x_0 < 0, \Delta P_1 < 0\big) > Pr\big(x_1 > 0 \mid x_0 < 0, \Delta P_1 > 0\big),$$

given that $\sigma_{\epsilon_1}\sigma_{z_1} < \hat{\sigma}_1$. In fact, this symmetry holds for all of our results and so, from now on, we will present them only from the perspective of an investor who holds a long position at date 0.

The above explanation of why informed traders may exhibit disposition effects also gives some indication of the investors' optimal reaction to price changes. When the information asymmetry between the two groups of investors sufficiently decreases from date 0 to date 1, informed traders find it optimal to sell shares on good news and buy them on bad news. In other words, informed investors follow a contrarian investment strategy. Their date 1 demand is negatively correlated with the price change ΔP_1.[17] Since this is true even if there are no noise traders, it follows that uninformed investors take the opposite position and, hence, optimally increase their stock holdings upon price appreciation, and decrease them upon price depreciation. That is to say, they rationally "chase the trend". In fact, the following corollary shows that this intuition is true whenever $\sigma_{\epsilon_1}\sigma_{z_1}$ is below the critical value $\hat{\sigma}_1$.

Corollary 1 *If $\sigma_{\epsilon_1}\sigma_{z_1} < \hat{\sigma}_1$, the trades of informed investors at date 1 are negatively correlated with the price change at date 1, whereas those of uninformed investors are positively correlated with ΔP_1, that is, $Cov(x_1, \Delta P_1) < 0$ and $Cov(y_1, \Delta P_1) > 0$.*

[17] Although the correlation between x_1 and ΔP_1 may seem a contemporaneous one, we believe the fact that investors can submit demands as a function of the current market price justifies our usage of the terms "contrarian" and "trend-chasing" with respect to these behaviors.

The above result is consistent with Brennan and Cao (1996) and Watanabe (2002). In these models, less informed investors tend to be trend-followers, even though they know they lose against better informed traders. It is worth noting that a recent study by Grinblatt and Keloharju (2000) seems to indicate that this result is not supported by the data. Using a comprehensive data set that documents the shareholdings of both institutional and retail investors in Finland from December 1994 to December 1996, Grinblatt and Keloharju report that the frequency of contrarian behavior seems to be inversely related to investor sophistication.[18] Furthermore, their analysis demonstrates that contrarian behavior is negatively correlated with investor performance. Investors who pursue momentum strategies outperform contrarian investors.[19] As we show in Section 4.2, however, these empirical findings are perfectly consistent with increasing information asymmetry between informed and uninformed investors in our model.

So far, we have only considered the frequency with which informed investors sell winning and losing stocks. We now turn our attention to the size of these trades. The following proposition shows that informed traders are not only more likely to sell winners than losers, they also sell larger amounts of their winning stocks than of their losing stocks.

Proposition 3 *If $\sigma_{\epsilon_1}\sigma_{z_1} < \hat{\sigma}_1$, the expected trade size is larger for sold winners than for sold losers, i.e.,*

$$E\big[|x_1| \mid x_0 > 0, \Delta P_1 > 0, x_1 < 0\big] > E\big[|x_1| \mid x_0 > 0, \Delta P_1 < 0, x_1 < 0\big].$$

In the limiting case when the variance of the date 1 supply shock goes to zero, the average trading volume for sold losers converges to zero. In other words, if the signal S_1 becomes public knowledge, informed investors never sell their losers to uninformed investors. The same is true for large values

[18] Grinblatt and Keloharju (2000) aggregate their data into six investor categories: households, nonprofit institutions, government, finance and insurance institutions, non-financial corporations, and foreign investors. They find that households, government investors, and nonprofit institutions exhibit contrarian behavior over all investment horizons, whereas finance and insurance institutions, as well as non-financial corporations, are only marginally contrarian. Foreign investors tend to be momentum investors (or trend-followers) over all horizons.

[19] Grinblatt and Keloharju (2000) measure performance by examining whether the buy ratio of future winning stocks exceeds the buy ratio of future losing stocks.

of σ_{z_1}. If the expected supply shock is large[20] (i.e., if there are many liquidity traders in the market), rational investors will always pursue a contrarian investment strategy in order to benefit from non-fundamental price fluctuations caused by liquidity traders. Numerical calculations show that the difference between the expected trading volume of winners and losers that are sold typically increases with σ_{z_1}. The empirical validity of Proposition 3 remains an open question, as none of the studies on the disposition effect that we are aware of reports the average trade size for realized winners and losers.

Whenever the aggregate stock supply varies over time and these variations are independent of the fundamentals, homogeneously informed investors will follow a contrarian strategy, and sell shares when the price is up and buy shares when the price is down. In such a symmetric information model, the period 2 price change $\Delta P_2 = V - P_1$ would on average be higher for losing stocks that investors hold on to than for winning stocks that investors decide to sell. In fact, this argument has been used in empirical studies (see, e.g., Odean (1998) and Brown et al. (2002)) to take the superior "ex-post return" of winners that are sold, compared to losers that are not sold, as clear evidence against an information-based explanation for the disposition effect. Odean concludes that the investors' preference for selling winners and holding on to losers "is not justified by subsequent portfolio performance." The following proposition shows that this conclusion is not always correct. Even if the expected future return of winners exceeds that of losers, investors might rationally sell their winners and hold on to their losers.

Proposition 4 *There exist parameter values such that informed investors rationally exhibit disposition effects, even though the expected period 2 (dollar) return of winning stocks that they sell exceeds that of losing stocks that they buy, i.e.,*

$$E\big[\Delta P_2 \mid x_0 > 0, \Delta P_1 > 0, x_1 < 0\big] > E\big[\Delta P_2 \mid x_0 > 0, \Delta P_1 < 0, x_1 > 0\big].$$

The economic intuition for this striking result is as follows. From the discussion above, we know that when the information asymmetry decreases sufficiently, uninformed investors "overreact"[21] to news released at date 1

[20]Note that $E[|z_t|] = \sqrt{2/\pi}\,\sigma_{z_t}$.

[21]From the standpoint of informed investors, the behavior of uninformed investors looks like an overreaction to the new information about V revealed through the price P_1. However, it is fully rational given their limited information $\mathcal{F}_1^U$.

from the perspective of informed investors. Thus, in contrast to Proposition 4, we would expect the average period 2 price change following bad news to be higher than that following good news. This effect, however, is mitigated and, if σ_{ϵ_1} and σ_{z_1} are sufficiently small, even reversed, by the fact that the premium required by risk-averse investors to absorb the supply shock z_0 is gradually reduced over time as investors become better informed. In other words, the negative effect a positive supply shock has on the price at date 0 is only partially reversed at date 1. If, in addition, the date 1 supply shock and/or the residual uncertainty are small, hence, causing only a minor price change that is going to be reversed at date 2, then the latter effect dominates, and the price changes ΔP_1 and ΔP_2 are positively correlated.[22]

To obtain a greater understanding of the above result, consider the case where the stock's payoff, as well as both signals, turn out to be equal to their expected value of zero, and assume that there is no supply shock at date 1. Since investors do not know this at date 0, they demand a risk premium for holding shares. Thus, if the stock supply z_0 is positive, the price at date 0 will be negative. This makes uninformed investors believe that informed investors have received bad news. When $S_1 = 0$ is revealed at date 1, uninformed investors will therefore adjust their expectations upward, whereas informed investors still expect the payoff to be zero. Moreover, since both types of investors now have more precise information about V, they bear less risk and consequently demand a lower risk premium. Thus, both groups of investors want to increase their portfolio holdings, but the change in demand is comparatively larger for uninformed traders. To clear the market, the price will go up until the amount of shares informed investors are willing to sell equals the amount uninformed investors want to buy.[23] But because of the residual uncertainty about V, this new price level will be below zero, the payoff informed investors expect to get at date 2. Thus, the period 2 price change will be positive as well. The reverse is true for a negative supply shock at date 0.[24] This shows that the supply shock z_0 causes the period 2 return, ΔP_2, to be negatively correlated with the informed investors' date 1 trades, x_1, but positively with the period 1 return, ΔP_1, and the investors' date 0 stock position, x_0. Therefore, in equilibrium, informed investors may

[22]This result is consistent with the momentum effect documented by Jegadeesh and Titman (1993). We further discuss the relation between price momentum and disposition effects in Section 5.

[23]This can also be seen from the price coefficients: $b < e$.

[24]Recall that for simplicity, we normalized the average stock supply to zero.

optimally reduce their period 2 stock holdings even though they anticipate a further price increase, and vice versa.[25]

The positive correlation of ΔP_2 with x_0 and ΔP_1 has implications for the period 2 return variability of winners and losers as well. The following proposition shows that the variance of the period 2 price change is typically higher for winners that informed investors sell at date 1 than for losers that they decide to keep. This result does not imply, however, that stocks with higher expected returns are more risky, from the investors' perspective. In fact, the variance of ΔP_2, conditional on the informed investors' date 1 information set, $\mathcal{F}_1^I$, is constant. The higher return variability of winners follows from the larger variation of the expected price change $E[\Delta P_2 \mid x_0, \Delta P_1, x_1]$ in the event $\{x_0 > 0, \Delta P_1 > 0, x_1 < 0\}$ ("sold winner"), caused by the positive correlation between the informed investors' initial stock holdings x_0 and the subsequent price change ΔP_1.

Proposition 5 *If σ_{z_1} is sufficiently small, the variance of ΔP_2 is higher for winning stocks that informed investors sell at date 1 than for losing stocks that they buy, i.e.,*

$$Var\big[\Delta P_2 \mid x_0 > 0, \Delta P_1 > 0, x_1 < 0\big] > Var\big[\Delta P_2 \mid x_0 > 0, \Delta P_1 < 0, x_1 > 0\big].$$

Although an analytical proof is available only when this result is stated for small σ_{z_1}, numerical calculations show that it generally holds whenever $\sigma_{\epsilon_1}\sigma_{z_1} < \hat{\sigma}_1$. In any event, Proposition 5 may provide us with a possible test of the model, as this result has yet to be confirmed empirically.

4.2 Trading behavior of uninformed investors

Using wealth, age, and trading experience as proxies for investor sophistication, recent empirical studies indicate that less sophisticated investors are more susceptible to disposition effects.[26] This result has generally been interpreted as evidence against rational explanations. Dhar and Zhu (2002), for

[25]Notice that Proposition 4 only makes a statement about the relationship between expected returns and *changes* in the informed investors' stock portfolios. In particular, it does not say that period 2 returns are negatively correlated with the investors' optimal date 1 stock holdings.

[26]See, e.g., Brown et al. (2002) and Dhar and Zhu (2002). In fact, Dhar and Zhu find that about one-fifth of the investors in their sample exhibit "reverse disposition effects". These are typically older and wealthier investors who classify themselves as professionals.

example, argue that, as investors gain more trading experience, they become aware of their own behavioral biases and adjust their investment strategies accordingly. In this section, we offer an alternative explanation. In particular, our analysis suggests that inexperienced investors rationally follow trading strategies, as predicted by the disposition effect, when the information asymmetry between investors increases (or, at least, does not decrease too much) prior to public announcements.[27]

Proposition 6 *Uninformed investors are more likely to sell their winning stocks than their losing stocks, i.e.,*

$$Pr\big(y_1 < 0 \mid y_0 > 0, \Delta P_1 > 0\big) > Pr\big(y_1 < 0 \mid y_0 > 0, \Delta P_1 < 0\big),$$

if and only if $\sigma_{\epsilon_1}\sigma_{z_1} > \hat{\sigma}_1$.

The intuition behind the above proposition is as follows. Suppose that the signal S_0 contains no information at all.[28] In this case, informed and uninformed investors hold identical portfolios at date 0, absorbing the supply shock z_0. At date 1, the stock price increases, if one of two things happens: (i) informed traders receive good news and, hence, revise their expectations about V upwards, or (ii) the aggregate stock supply drops. Not being able to distinguish these two possible causes, uninformed investors respond to the observed price increase by selling (some of) their shares. The reverse is true if the price drops at date 1. In this case, uninformed investors optimally buy shares, hoping that the price depreciation is due to a positive supply shock and not to bad news about V. This analysis shows that, unlike the case where S_1 is announced publicly, uninformed investors now pursue a contrarian investment strategy.

Corollary 2 *If* $\sigma_{\epsilon_1}\sigma_{z_1} > \hat{\sigma}_1$*, the date 1 trades of uninformed investors are negatively correlated with the date 1 price change, i.e.,* $Cov(y_1, \Delta P_1) < 0$.

In the previous section, we argued that the risk premium required to bear the supply shock z_0 gradually decreases over time as investors become better

[27] The investors' information acquisition decision is exogenous to our model. However, since less experienced traders face higher information production (or processing) costs, it seems reasonable that these traders will be less informed.

[28] In this case, the condition in Proposition 6 is satisfied for all $\sigma_{\epsilon_1}, \sigma_{z_1} \in \mathbb{R}_+$, because $\hat{\sigma}_1$ converges to zero as σ_{ϵ_0} goes to infinity.

informed. In the absence of a date 1 supply shock, this positive autocorrelation in the conditional risk premium is sufficient to cause price continuation, which is a necessary condition for the period 2 return of winners to exceed that of losers (Proposition 4). In the case of a non-zero date 1 supply shock, the positive serial correlation in returns is reduced by the fact that the conditional risk premium attributable to the shock z_1 reverses by date 2. Still, if σ_{z_1} is small, the gradual decrease of the risk premium related to z_0 dominates the price reversal due to z_1, making the overall serial correlation positive. However, as pointed out above, a sufficiently large supply shock at date 1 is essential for P_1 not to convey too much information, so that uninformed investors prefer to sell winners and buy losers, hoping to benefit from liquidity trades. In fact, there exists a non-empty set of parameter values for which both effects are present: (i) the information revealed by P_1 is noisy enough to make uninformed investors follow a contrarian strategy, and (ii) the reversal of the conditional risk premium due to z_1 is small enough to cause a positive autocorrelation in price changes. As a result, we obtain the empirically observed phenomenon that uninformed investors sell their winning stocks even though the subsequent return of these stocks is on average higher than that of losing stocks that they keep in their portfolios. Further, for this set of parameter values, the variance of ΔP_2 of the winning stocks that are sold is higher than that of the losing stocks that are not sold. These results are summarized in the following proposition.

Proposition 7 *There exists a $\sigma_1^* > \hat{\sigma}_1$ such that for all $\sigma_{\epsilon_1}\sigma_{z_1} \in (\hat{\sigma}_1, \sigma_1^*)$, the expected period 2 (dollar) return of winning stocks that uninformed investors sell is higher than that of losing stocks that they buy, i.e.,*

$$E\big[\Delta P_2 \mid y_0 > 0, \Delta P_1 > 0, y_1 < 0\big] > E\big[\Delta P_2 \mid y_0 > 0, \Delta P_1 < 0, y_1 > 0\big].$$

Further, the variance of ΔP_2 is higher for winning stocks that are sold than for losing stocks that are not sold, i.e.,

$$Var\big[\Delta P_2 \mid y_0 > 0, \Delta P_1 > 0, y_1 < 0\big] > Var\big[\Delta P_2 \mid y_0 > 0, \Delta P_1 < 0, y_1 > 0\big],$$

if $\hat{\sigma}_1 < \sigma_{\epsilon_1}\sigma_{z_1} < \bar{\sigma}_1$, where

$$\bar{\sigma}_1 = \sqrt{\frac{M\,\sigma_{\epsilon_0}^2\sigma_{z_0}^2\left(M + \gamma^2\sigma_{\epsilon_0}^2\sigma_{z_0}^2\right)}{M^2\left(1+\sigma_{\epsilon_0}^2\right) + \gamma^2\sigma_{\epsilon_0}^2\sigma_{z_0}^2\left(M+\sigma_{\epsilon_0}^2\right)}}.$$

4.3 Discussion

Our analysis shows that disposition effects arise quite naturally in a world with changing information asymmetry. However, some caution is required in interpreting our results. In particular, our assumption that the aggregate stock supply is subject to stochastic changes deserves more attention. While some kind of "noise" is necessary to prevent prices from fully revealing the investors' private information, these supply shocks also introduce a bias to the PGR/PLR ratio. The larger the expected supply shock, the more profitable it is for investors to follow a contrarian investment strategy.[29]

This bias is especially troubling when we think of supply shocks as being the result of liquidity trading activity, for it then means that we ignore the trades of a potentially large group of investors that are obviously negatively correlated with prices: liquidity traders tend to buy stocks when prices are up and to sell stocks when prices are down, behavior that clearly contradicts the disposition effect. In order to assess the severity of this bias, we recalculate our PGR and PLR measures, only taking into account the *information-based component* of the investors' demand, defined as $\tilde{x}_t = x_t - z_t$ and $\tilde{y}_t = y_t - z_t$.[30] This alternate definition ensures that the aggregate demand of informed and uninformed investors is uncorrelated with price changes. However, it makes our analysis less tractable. Necessary and sufficient conditions for informed and uninformed investors to exhibit disposition effects are no longer available in closed form. Nevertheless, numerical computations show that our basic results remain unchanged. Table 1 reports the PGR/PLR ratio and the expected period 2 return of winners and losers for two sets of parameters, one that decreases the information asymmetry between informed and uninformed traders across dates 0 and 1 (panel A), and one that increases it (panel B). As expected, in the former case, informed investors show disposition effects (PGR/PLR ratio > 1), whereas in the latter case uninformed investors do. For comparison, we calculate the disposition statistics for the investors' total demand (reported in parentheses). These examples show that the intuition behind our results generalizes to a more realistic setting with constant aggre-

[29]Note that the investors' PGR/PLR ratio can be upwardly biased as well as downwardly biased, depending on the magnitude of the date 1 supply shock relative to that of the date 0 shock.

[30]This is equivalent to assuming that investors experience endowment shocks of z_0 and z_1 at dates 0 and 1, respectively, but cannot infer the aggregate stock supply from their own endowment.

Panel A: Decreasing information asymmetry		
	informed investors	uninformed investors
PGR/PLR ratio	2.493 (1.978)	0.401 (0.000)
exp. ΔP_2 difference	0.057 (-0.066)	-0.455 (-1.119)

Panel B: Increasing information asymmetry		
	informed investors	uninformed investors
PGR/PLR ratio	0.918 (1.196)	1.089 (∞)
exp. ΔP_2 difference	0.075 (-0.110)	0.836 (0.222)

Table 1: PGR/PLR ratios and differences in expected period 2 returns between winners that are sold and losers that are not sold for two sets of parameters: $\gamma = 1$, $M = \frac{1}{2}$, $\sigma_{\epsilon_0} = 1$, $\sigma_{\epsilon_1} = \frac{1}{2}$, $\sigma_{z_0} = 2$, $\sigma_{z_1} = \frac{1}{2}$ in Panel A, and $\gamma = 1$, $M = \frac{1}{2}$, $\sigma_{\epsilon_0} = 3$, $\sigma_{\epsilon_1} = \frac{1}{2}$, $\sigma_{z_0} = 2$, $\sigma_{z_1} = \frac{1}{2}$ in Panel B. Values in parentheses are based on total demand.

gate stock supply. Interestingly, the difference between the expected period 2 return of winning stocks that are sold and that of losing stocks that are not sold increases when we consider only the information-based component of the investors' demand. The reason is that a positive date 1 price change is likely to be accompanied by a negative supply shock z_1, which implies that the information-based demand $\tilde{x}_1$ will on average be higher than the total demand x_1 in this case. Thus, even better news is necessary to convince investors to sell some of their shares (i.e., to ensure that $\tilde{x}_1 < 0$). The opposite is true for a negative date 1 price change.

5 Momentum

The consistent profitability of momentum strategies, i.e., strategies that buy stocks that performed well in the past and sell stocks that performed poorly, remains one of the most puzzling anomalies in finance. Jegadeesh and Titman (1993) show that past winners continue to outperform past losers by about 1% per month over the subsequent three to 12 months. Rouwenhorst (1998) finds similar results in 12 European markets. Recent empirical evidence indicates that momentum profits cannot be explained by Fama-French factors, industry effects, or cross-sectional differences in expected returns.[31]

Various explanations for the superior performance of momentum strategies have been advanced, most of them appealing to behavioral biases that lead investors to either underreact or belatedly overreact to information.[32] Johnson (2002) offers a rational explanation for these momentum effects. He shows that a single-firm model with a standard pricing kernel can produce a strong positive correlation between past realized returns and current expected returns when expected dividend growth rates vary over time. A different approach is taken by Holden and Subrahmanyam (2002), who show that medium-term continuations can arise in a rational expectations model when private information is received sequentially by risk-averse agents. In this case, the risk borne by the market decreases over time. This effect causes a positive autocorrelation in the conditional risk premium and thus leads to price continuations.

As pointed out in Section 4, the same effect is at work in our model.[33] The risk premium related to the supply shock z_0 decreases across dates 0 and 1 as (some) investors observe the signal S_1, and it further decreases across dates 1 and 2 as prices approach full revelation. This gradual decrease introduces persistence into price changes. On the other hand, the additional risk premium due to the shock z_1 at date 1 reverses by date 2, causing a price reversal, on average. If this additional date 1 risk premium is small — either because the average date 1 supply shock is small, or the residual uncertainty is low (which is the case when the mass of informed investors is large, the noise variance of S_1 is small, and/or the variance of the date 1 supply shock is small) — the first effect dominates, resulting in positive serial correlation in returns.

Proposition 8 *The price changes $\Delta P_1 = P_1 - P_0$ and $\Delta P_2 = V - P_1$ are positively correlated if and only if $\sigma_{\epsilon_1}\sigma_{z_1} < \bar{\sigma}_1$, where $\bar{\sigma}_1$ is defined in Proposition 7.*

Grinblatt and Han (2001) link price momentum to the disposition effect. In their model, a fraction of homogeneously informed investors is assumed to

[31]See Fama and French (1996), Moskowitz and Grinblatt (1999), Grundy and Martin (2001), and Jegadeesh and Titman (2001, 2002).

[32]See, e.g., Barberis, Shleifer, and Vishny (1998), Daniel, Hirshleifer, and Subrahmanyam (1998), and Hong and Stein (1999).

[33]In fact, the two-period model of Holden and Subrahmanyam (2002) corresponds to the special case $\sigma_{\epsilon_0} = \infty$ of our model.

have a higher demand for losing stocks than for winning stocks, other things being equal. Grinblatt and Han show that such a demand perturbation tends to generate short-term price underreaction to public information and, hence, price continuation. This is in stark contrast to our result. While Grinblatt and Han take the trading strategies of "disposition investors" as given, and show that such a behavioral bias can cause momentum, we demonstrate that both disposition effects and momentum can arise in a world with fully rational agents. In fact, since $\hat{\sigma}_1 < \bar{\sigma}_1$, our model shows that both scenarios — informed investors exhibiting disposition effects, and uninformed investors exhibiting disposition effects — are consistent with price continuation. However, there is no causal relation between these two effects. When the expected date 1 supply shock is sufficiently large, both informed and uninformed investors will sell their winners and hold on to their losers, even though — or rather because — price changes are negatively correlated. However, as discussed in the previous section, price momentum is a necessary condition for the average ex-post returns of winning stocks that investors sell to be higher than those of losing stocks that they hold on to (Propositions 4 and 7).

Further empirical implications concerning the relation between momentum and the disposition effect can be obtained through numerical computations. Not surprisingly, we find that the correlation between ΔP_1 and ΔP_2 is decreasing in σ_{z_1}, and thus, is generally higher for values of $\sigma_{\epsilon_1}\sigma_{z_1}$ in the interval $(0, \hat{\sigma}_1)$ than in the interval $(\hat{\sigma}_1, \bar{\sigma}_1)$. This implies that the disposition behavior of less informed investors is concentrated primarily in relation to stocks with weak momentum.[34] Put differently, the propensity of less informed investors to sell winners and hold on to losers, measured by the difference between the ratios PGR and PLR, is inversely related to the persistence in stock returns.

Indirect empirical support for this result comes from a recent study by Ranguelova (2001). She documents that disposition behavior is more pronounced for large-cap stocks. In fact, trades in stocks at the bottom 40 % of the market capitalization distribution exhibit a "reverse disposition effect": investors keep their winners and sell their losers. While difficult to explain with prospect-theory type preferences, these findings are consistent with our information-based explanation, since price continuations are typically weaker

[34] We cannot make an analogous statement for (better) informed investors, because they exhibit disposition behavior for small values of σ_{z_1}, as well as for large values of σ_{z_1} (the condition $\sigma_{\epsilon_1}\sigma_{z_1} < \hat{\sigma}_1$ is sufficient but not necessary). In the former case, they primarily benefit from uninformed trend-followers, in the latter case from supply shocks.

for large firms (see, e.g., Jegadeesh and Titman (1993) or Hong, Lim, and Stein (2000)).

6 Conclusion

This paper shows that rational investors can exhibit disposition effects even in a world without taxes, transaction costs, and portfolio rebalancing needs. We present a dynamic rational expectations model of asset prices that allows us to derive the investors' optimal trading strategies in closed form. Our focus is on analyzing how dynamic changes in information asymmetry influence the investors' behavior. We find that better informed investors tend to sell their winners and to hold on to their losers when the information asymmetry sharply decreases prior to public news releases (such as earnings announcements or annual reports). Less informed investors, on the other hand, exhibit such behavior when the information asymmetry increases or moderately decreases over time. In addition, our results show that, on average, investors sell larger amounts of their winning stocks than of their losing stocks.

Interestingly, investors may optimally pursue a contrarian investment strategy even though past winners continue to outperform past losers in the future. The reason is that the increase in an investor's stock demand following good news can be outweighed by an opposite effect due to changes in her information quality (relative to that of other investors). If informational differences between investors decrease over time, better informed investors will trade less aggressively on new information, compared to less informed investors, and, hence, will optimally reduce their stock holdings when they receive good news and increase them when they receive bad news. The opposite is true when the information asymmetry between investors increases over time. This result indicates that existing empirical tests rejecting an information-based explanation for the disposition effect are inconclusive.

We also study the relationship between optimal trading strategies and equilibrium price dynamics. We find that disposition behavior is generally consistent with both price continuations and price reversals. However, for ex-post returns of winning stocks that investors sell to exceed those of losing stocks that they hold on to, price changes must be positively correlated. Further, our model predicts that the disposition behavior of less informed investors is concentrated primarily in relation to stocks with weak momentum. In contrast to Grinblatt and Han (2001), who take the disposition behavior of a group of investors as given, and show that such a behavioral bias can cause price momentum, our model suggests that both effects can be the result of informational differences between rational investors.

Of course, investors may also exhibit prospect theory preferences, or irrationally believe in mean-reverting prices. By pointing out that disposition effects are not intrinsically at odds with rational behavior, this paper simply shows that the existing empirical evidence is insufficient to rule out either explanation.

7 Appendix

The following lemma is a standard result on multivariate normal random variables (see, e.g., Bray (1981) or Brunnermeier (2001), chapter 3) and is used to derive the optimal date 0 demand of investors:

Lemma 3 *Let $\boldsymbol{x} \in \mathbb{R}^n$ be a normally distributed random vector with mean (vector) $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$. Then,*

$$E\left[e^{\boldsymbol{x}^T \boldsymbol{A}\,\boldsymbol{x} + \boldsymbol{b}^T \boldsymbol{x} + c}\right] = |\boldsymbol{I} - 2\boldsymbol{\Sigma}\boldsymbol{A}|^{-\frac{1}{2}} \exp\left\{ c + \boldsymbol{b}^T \boldsymbol{\mu} + \boldsymbol{\mu}^T \boldsymbol{A}\,\boldsymbol{\mu} + \frac{1}{2}(\boldsymbol{b} + 2\boldsymbol{A}\,\boldsymbol{\mu})^T (\boldsymbol{\Sigma}^{-1} - 2\boldsymbol{A})^{-1} (\boldsymbol{b} + 2\boldsymbol{A}\,\boldsymbol{\mu}) \right\} \quad (7)$$

where $\boldsymbol{A}$ is a symmetric $n \times n$ matrix, $\boldsymbol{b}$ is an n-vector, and c is a scalar.

Note that (7) is only well-defined if $\boldsymbol{I} - 2\boldsymbol{\Sigma}\boldsymbol{A}$ is positive definite.

Further, in order to calculate the conditional probabilities and expectations in Section 4, we use the following results on normally distributed random variables (see David (1953) and Kamat (1953)):

Lemma 4 *Let $\{x_i\}_{i=1,2,3}$ be jointly normally distributed random variables with mean zero and correlation $\{\rho_{ij}\}_{i,j=1,2,3}$. Then,*

$$Pr(x_1 > 0, x_2 > 0) = \frac{1}{2\pi}(\pi - \arccos \rho_{12})$$

and

$$Pr(x_1 > 0, x_2 > 0, x_3 > 0) = \frac{1}{4\pi}(2\pi - \arccos \rho_{12} - \arccos \rho_{13} - \arccos \rho_{23}).$$

Lemma 5 *Let x_1 and x_2 be jointly normally distributed random variables with mean zero, variance one, and correlation ρ, and let $[m,n]$ denote the "incomplete moment" given by*

$$[m,n] = \int_0^\infty \int_0^\infty x_1^m \, x_2^n \, f(x_1,x_2)\, dx_1 dx_2,$$

where $f(x_1,x_2)$ denotes the joint probability-density function. Then,

$$\begin{aligned}
[1,0] &= \frac{1+\rho}{2\sqrt{2\pi}}, \\
[2,0] &= \frac{1}{2\pi}\left(\pi - \arccos\rho + \rho\sqrt{1-\rho^2}\right), \\
[1,1] &= \frac{1}{2\pi}\left(\rho(\pi - \arccos\rho) + \sqrt{1-\rho^2}\right).
\end{aligned}$$

For incomplete moments of the trivariate normal distribution, we refer the reader to Kamat (1953).

The following corollary will be useful for proving Propositions 5 and 7.

Corollary 3 *Let x_1 and x_2 be jointly normally distributed random variables with mean zero and correlation ρ. Then,*

$$Var\big[x_1 + x_2 \mid x_1 > 0, x_2 < 0\big] > Var\big[x_1 + x_2 \mid x_1 > 0, x_2 > 0\big] \tag{8}$$

for all $\rho \in (-1,0)$.

Proof: Let ΔVar denote the difference between the two conditional variances in (8), i.e.,

$$\Delta Var = Var\big[x_1 + x_2 \mid x_1 > 0, x_2 < 0\big] - Var\big[x_1 + x_2 \mid x_1 > 0, x_2 > 0\big],$$

and recall that

$$Var\big[x_1 + x_2 \mid \cdot\big] = Var\big[x_1 \mid \cdot\big] + 2\,Cov\big[x_1, x_2 \mid \cdot\big] + Var\big[x_2 \mid \cdot\big].$$

From Lemmas 4 and 5, we have

$$\begin{aligned}
&Var\big[x_i \mid x_1 > 0, x_2 < 0\big] \\
&\quad = E\big[x_i^2 \mid x_1 > 0, x_2 < 0\big] - \big(E\big[x_i \mid x_1 > 0, x_2 < 0\big]\big)^2 \\
&\quad = \left(1 - \frac{\rho\sqrt{1-\rho^2}}{\pi - \arccos(-\rho)} - \frac{\pi(1-\rho)^2}{2(\pi - \arccos(-\rho))^2}\right) Var[x_i]
\end{aligned}$$

and

$$
\begin{aligned}
&Var\big[x_i \mid x_1 > 0, x_2 > 0\big] \\
&\qquad = \quad E\big[x_i^2 \mid x_1 > 0, x_2 > 0\big] - \big(E\big[x_i \mid x_1 > 0, x_2 > 0\big]\big)^2 \\
&\qquad = \quad \left(1 + \frac{\rho\sqrt{1-\rho^2}}{\pi - \arccos\rho} - \frac{\pi(1+\rho)^2}{2(\pi - \arccos\rho)^2}\right) Var[x_i],
\end{aligned}
$$

for $i = 1, 2$. Similarly, the conditional covariances are given by

$$
\begin{aligned}
&Cov\big[x_1, x_2 \mid x_1 > 0, x_2 < 0\big] \\
&\quad = \; E\big[x_1 x_2 \mid x_1 > 0, x_2 < 0\big] - E\big[x_1 \mid x_1 > 0, x_2 < 0\big] E\big[x_2 \mid x_1 > 0, x_2 < 0\big] \\
&\quad = \; \left(\rho - \frac{\sqrt{1-\rho^2}}{\pi - \arccos(-\rho)} + \frac{\pi(1-\rho)^2}{2(\pi - \arccos(-\rho))^2}\right) \sqrt{Var[x_1]Var[x_2]}
\end{aligned}
$$

and

$$
\begin{aligned}
&Cov\big[x_1, x_2 \mid x_1 > 0, x_2 > 0\big] \\
&\quad = \; E\big[x_1 x_2 \mid x_1 > 0, x_2 > 0\big] - E\big[x_1 \mid x_1 > 0, x_2 > 0\big] E\big[x_2 \mid x_1 > 0, x_2 > 0\big] \\
&\quad = \; \left(\rho + \frac{\sqrt{1-\rho^2}}{\pi - \arccos\rho} - \frac{\pi(1+\rho)^2}{2(\pi - \arccos\rho)^2}\right) \sqrt{Var[x_1]Var[x_2]}.
\end{aligned}
$$

Putting all terms together, we can rewrite ΔVar as follows:

$$
\Delta Var = \frac{c_1^2\, Var[x_1] - 2\, c_{12}\, \sqrt{Var[x_1]Var[x_2]} + c_2^2\, Var[x_2]}{\frac{2}{\pi}(\pi - \arccos\rho)^2(\arccos\rho)^2}
$$

where

$$
\begin{aligned}
c_1^2 = c_2^2 &= \rho\,\pi^2 - 2\,\rho\sqrt{1-\rho^2}(\pi - \arccos\rho)(\arccos\rho) \\
&\quad -2\,(\arcsin\rho)\big(\pi(1+\rho^2) - 2\,\rho(\arcsin\rho)\big), \\
c_{12} &= -\tfrac{1}{2}(1+\rho^2)\,\pi^2 + 2\,\sqrt{1-\rho^2}(\pi - \arccos\rho)(\arccos\rho) \\
&\quad +2\,(\arcsin\rho)\big(2\,\rho\,\pi - (1+\rho^2)(\arcsin\rho)\big).
\end{aligned}
$$

Since $c_{12} < c_1\, c_2 = c_1^2$ for all $\rho \in (-1, 0)$, ΔVar is strictly positive, if x_1 and x_2 are negatively correlated. $\square$

Conditional moments of V

The expected value and the variance of V conditional on the informed investors' date 1 information set, $\mathcal{F}_1^I$, are given by

$$\begin{aligned} E\left[V \mid \mathcal{F}_1^I\right] &= E[V \mid S_0, S_1] \\ &= \underbrace{\frac{\sigma_{\epsilon_1}}{\sigma_{\epsilon_0}+\sigma_{\epsilon_1}+\sigma_{\epsilon_0}\sigma_{\epsilon_1}}}_{\equiv\lambda_0} S_0 + \underbrace{\frac{\sigma_{\epsilon_0}}{\sigma_{\epsilon_0}+\sigma_{\epsilon_1}+\sigma_{\epsilon_0}\sigma_{\epsilon_1}}}_{\equiv\lambda_1} S_1, \quad (9) \\ Var\left[V \mid \mathcal{F}_1^I\right] &= Var[V \mid S_0, S_1] = \frac{\sigma_{\epsilon_0}\sigma_{\epsilon_1}}{\sigma_{\epsilon_0}+\sigma_{\epsilon_1}+\sigma_{\epsilon_0}\sigma_{\epsilon_1}} \equiv \tau_I. \quad (10) \end{aligned}$$

Similarly, the conditional moments of V based on the uninformed investors' date 1 information set, $\mathcal{F}_1^U$, are given by

$$E\left[V \mid \mathcal{F}_1^U\right] = \underbrace{\frac{a\,\sigma_{P_1}^2 - (c+d)\,\sigma_{P_0,P_1}}{\sigma_{P_0}^2\sigma_{P_1}^2 - \sigma_{P_0,P_1}^2}}_{\equiv\kappa_0} P_0 + \underbrace{\frac{(c+d)\,\sigma_{P_0}^2 - a\,\sigma_{P_0,P_1}}{\sigma_{P_0}^2\sigma_{P_1}^2 - \sigma_{P_0,P_1}^2}}_{\equiv\kappa_1} P_1, \quad (11)$$

$$Var\left[V \mid \mathcal{F}_1^U\right] = 1 - \frac{a^2\,\sigma_{P_1}^2 + (c+d)^2\,\sigma_{P_0}^2 - 2\,a\,(c+d)\,\sigma_{P_0,P_1}}{\sigma_{P_0}^2\sigma_{P_1}^2 - \sigma_{P_0,P_1}^2} \equiv \tau_U, \quad (12)$$

where

$$\begin{aligned} \sigma_{P_0}^2 &\equiv Var[P_0] = a^2(1+\sigma_{\epsilon_0}^2) + b^2\sigma_{z_0}^2, \quad (13) \\ \sigma_{P_1}^2 &\equiv Var[P_1] = c^2(1+\sigma_{\epsilon_0}^2) + d^2(1+\sigma_{\epsilon_1}^2) + 2\,c\,d + e^2\sigma_{z_0}^2 + f^2\sigma_{z_1}^2, \quad (14) \\ \sigma_{P_0,P_1} &\equiv Cov[P_0,P_1] = a\,c(1+\sigma_{\epsilon_0}^2) + a\,d + b\,e\,\sigma_{z_0}^2. \quad (15) \end{aligned}$$

Proof of Lemma 1

Let us define $\mathcal{J}_I(x_0)$ to be an informed trader's maximum expected utility from date 2 consumption given that her date 0 stock holding is x_0, that is,

$$\mathcal{J}_I(x_0) = \max_{x_1}\; E\left[-\exp\left\{-\gamma\left(x_0(V-P_0) + x_1(V-P_1)\right)\right\} \mid \mathcal{F}_1^I\right]. \quad (16)$$

At date 0, each informed investor faces the following optimization problem:

$$\max_{x_0}\; E\left[\mathcal{J}_I(x_0) \mid \mathcal{F}_0^I\right], \quad (17)$$

where the investor's date 0 information set is given by $\mathcal{F}_0^I = \{S_0, P_0, z_0\}$. Substituting the investor's optimal date 1 demand given by (3) into equation (16), we can rewrite the investor's date 1 value function as

$$\begin{aligned}
\mathcal{J}_I(x_0) &= -\exp\left\{-\gamma\left(x_0(P_1 - P_0) + \frac{(\lambda_0 S_0 + \lambda_1 S_1 - P_1)^2}{2\,\gamma\,\tau_I}\right)\right\} \\
&= -\exp\left\{\gamma P_0 x_0 - \frac{1}{2\tau_I}\lambda_0^2 S_0^2 - \underbrace{\begin{pmatrix} \gamma x_0 - \frac{1}{\tau_I}\lambda_0 S_0 \\ \frac{1}{\tau_I}\lambda_0\lambda_1 S_0 \end{pmatrix}^T}_{\equiv \boldsymbol{b}_I^T} \begin{pmatrix} P_1 \\ S_1 \end{pmatrix}\right. \\
&\qquad \left. - \begin{pmatrix} P_1 \\ S_1 \end{pmatrix}^T \underbrace{\frac{1}{2\tau_I}\begin{pmatrix} 1 & -\lambda_1 \\ -\lambda_1 & \lambda_1^2 \end{pmatrix}}_{\equiv \boldsymbol{A}_I} \begin{pmatrix} P_1 \\ S_1 \end{pmatrix}\right\}. \qquad (18)
\end{aligned}$$

Given the conjectured equilibrium price function in (2), P_1 and S_1 are jointly normally distributed with respect to the date 0 information set, $\mathcal{F}_0^I$. Thus, it follows from Lemma 3 that maximizing (17) with respect to x_0 is equivalent to maximizing

$$\boldsymbol{b}_I^T \boldsymbol{\mu}_I - \frac{1}{2}(\boldsymbol{b}_I + 2\boldsymbol{A}_I\,\boldsymbol{\mu}_I)^T \underbrace{(\boldsymbol{\Sigma}_I^{-1} + 2\boldsymbol{A}_I)^{-1}}_{\equiv \boldsymbol{G}_I}(\boldsymbol{b}_I + 2\boldsymbol{A}_I\,\boldsymbol{\mu}_I) - \gamma\, P_0\, x_0, \qquad (19)$$

where $\boldsymbol{\mu}_I$ denotes the expectation and $\boldsymbol{\Sigma}_I$ the variance of the random vector $(P_1, S_1)^T$, conditional on $\mathcal{F}_0^I$:

$$\boldsymbol{\mu}_I \equiv E\left[\begin{pmatrix} P_1 \\ S_1 \end{pmatrix} \Big| \mathcal{F}_0^I\right] = \begin{pmatrix} \left(c + \frac{d}{1+\sigma_{\epsilon_0}^2}\right) S_0 + e\, z_0 \\ \frac{1}{1+\sigma_{\epsilon_0}^2} S_0 \end{pmatrix}, \qquad (20)$$

$$\begin{aligned}
\boldsymbol{\Sigma}_I &\equiv Var\left[\begin{pmatrix} P_1 \\ S_1 \end{pmatrix} \Big| \mathcal{F}_0^I\right] \\
&= \begin{pmatrix} \frac{d^2(\sigma_{\epsilon_0}^2 + \sigma_{\epsilon_1}^2 + \sigma_{\epsilon_0}^2\sigma_{\epsilon_1}^2)}{1+\sigma_{\epsilon_0}^2} + f^2\sigma_{z_1}^2, & d\left(\frac{\sigma_{\epsilon_0}^2}{1+\sigma_{\epsilon_0}^2} + \sigma_{\epsilon_1}^2\right) \\ d\left(\frac{\sigma_{\epsilon_0}^2}{1+\sigma_{\epsilon_0}^2} + \sigma_{\epsilon_1}^2\right), & \frac{\sigma_{\epsilon_0}^2}{1+\sigma_{\epsilon_0}^2} + \sigma_{\epsilon_1}^2 \end{pmatrix}. \qquad (21)
\end{aligned}$$

Substituting the expressions for $\boldsymbol{A}_I$ and $\boldsymbol{b}_I$ from (18) into (19), we derive the first-order condition for a maximum with respect to x_0 as

$$x_0 = \frac{E\left[P_1 \mid \mathcal{F}_0^I\right] - P_0}{\gamma\, G_{11}^I} + \frac{G_{11}^I - \lambda_1\, G_{12}^I}{G_{11}^I}\,\frac{\lambda_0 S_0 + \lambda_1 E\left[S_1 \mid \mathcal{F}_0^I\right] - E\left[P_1 \mid \mathcal{F}_0^I\right]}{\gamma\,\tau_I}$$

$$= \frac{E\left[P_1 \mid \mathcal{F}_0^I\right] - P_0}{\gamma\, G_{11}^I} + \frac{G_{11}^I - \lambda_1\, G_{12}^I}{G_{11}^I}\, \frac{E\left[V - P_1 \mid \mathcal{F}_0^I\right]}{\gamma\, \tau_I} \tag{22}$$

where G_{ij}^I are the elements of the matrix $\boldsymbol{G}_I$. It is easily verified that the demand defined by (22) is the unique maximum, since (19) is strictly concave in x_0.[35] □

Proof of Lemma 2

The optimal date 0 stock holdings of uninformed investors are found by solving the problem

$$\max_{y_0}\; E\left[\mathcal{J}_U(y_0) \mid \mathcal{F}_0^U\right], \tag{23}$$

where $\mathcal{J}_U(y_0)$ is the uninformed investors' date 1 value function:

$$\begin{aligned} \mathcal{J}_U(y_0) &= \max_{y_1}\; E\left[-\exp\left\{-\gamma\left(y_0(V - P_0) + y_1(V - P_1)\right)\right\} \mid \mathcal{F}_1^U\right] \\ &= -\exp\left\{-\gamma\left(y_0(P_1 - P_0) + \frac{(\kappa_0 P_0 + (\kappa_1 - 1)P_1)^2}{2\,\gamma\,\tau_U}\right)\right\}. \end{aligned}$$

The only random variable in (23) is the date 1 price P_1, which is normally distributed given $\mathcal{F}_0^U$. Therefore, we can use Lemma 3 to rewrite the uninformed investors' objective as

$$\max_{y_0}\; \gamma(\mu_U - P_0)\, y_0 - \frac{1}{2}\, G_U \left(\gamma y_0 + \frac{\kappa_1 - 1}{\tau_U}(\kappa_0 P_0 + (\kappa_1 - 1)\mu_U)\right)^2,$$

where G_U is a positive constant equal to $(1/\Sigma_U + (\kappa_1 - 1)^2/\tau_U)^{-1}$ and μ_U and Σ_U denote the conditional expectation and variance of P_1:

$$\mu_U \;\equiv\; E\left[P_1 \mid \mathcal{F}_0^U\right] = \frac{\sigma_{P_0,P_1}}{\sigma_{P_0}^2}\, P_0, \tag{24}$$

$$\Sigma_U \;\equiv\; Var\left[P_1 \mid \mathcal{F}_0^U\right] = \sigma_{P_1}^2 - \frac{\sigma_{P_0,P_1}^2}{\sigma_{P_0}^2}, \tag{25}$$

[35] The second derivative of (19) is equal to $-\gamma^2\, G_{11}^I$. If we let $\sigma_{S_1}^2$ denote the variance of S_1, $\sigma_{P_1}^2$ the variance of P_1, and ρ_{P_1,S_1} the correlation between P_1 and S_1, conditional on $\mathcal{F}_0^I$, we can rewrite G_{11}^I as follows:

$$G_{11}^I = \frac{\lambda_1^2(1 - \rho_{P_1,S_1}^2)\sigma_{P_1}^2\sigma_{S_1}^2 + \tau_I\sigma_{S_1}^2}{\lambda_1^2\sigma_{P_1}^2 - 2\lambda_1\rho_{P_1,S_1}\sigma_{P_1}\sigma_{S_1} + \sigma_{S_1}^2 + \tau_I}$$

This expression clearly shows that G_{11}^I is strictly positive and, hence, that (19) is a concave function of x_0.

where $\sigma^2_{P_0}$, $\sigma^2_{P_1}$, and σ_{P_0,P_1} are defined in (13) – (15). The unique[36] optimum of this quadratic maximization problem is given by

$$\begin{aligned} y_0 &= \frac{E\left[P_1 \mid \mathcal{F}_0^U\right] - P_0}{\gamma\, G_U} + \frac{1-\kappa_1}{G_U}\, \frac{\kappa_0 P_0 + (\kappa_1 - 1) E\left[P_1 \mid \mathcal{F}_0^U\right]}{\gamma\, \tau_U} \\ &= \frac{E\left[P_1 \mid \mathcal{F}_0^U\right] - P_0}{\gamma\, G_U} + \frac{1-\kappa_1}{G_U}\, \frac{E\left[V - P_1 \mid \mathcal{F}_0^U\right]}{\gamma\, \tau_U} \end{aligned}$$

□

Proof of Theorem 1

To prove that the linear price functions specified in Theorem 1 constitute a rational expectations equilibrium, it only remains to show that they clear the market for all possible realizations of S_0, S_1, z_0, and z_1.

The market-clearing condition at date 0 is given by

$$M\, x_0 + (1-M)\, y_0 = z_0.$$

Substituting the equilibrium price coefficients *a, b, c, d, e,* and *f* into the expressions for the conditional moments $\boldsymbol{\mu}_I$, $\boldsymbol{\Sigma}_I$, μ_U, and Σ_U given by (20), (21), (24), and (25), we can rewrite the investors' date 0 demand functions as follows:

$$x_0 = \frac{1}{\gamma}\left(\frac{1}{\sigma^2_{\epsilon_0}}\, S_0 - \left(1 + \frac{1}{\sigma^2_{\epsilon_0}}\right)\, P_0\right), \tag{26}$$

$$y_0 = -\frac{\gamma\, \sigma^2_{\epsilon_0}\, \sigma^2_{z_0}}{M + \gamma^2\, \sigma^2_{\epsilon_0}\, \sigma^2_{z_0}}\, P_0. \tag{27}$$

Thus, the stock market clears at date 0, if

$$\frac{M}{\gamma\, \sigma^2_{\epsilon_0}}\left(S_0 - \left(1 + \sigma^2_{\epsilon_0}\right)\, P_0\right) - \frac{(1-M)\, \gamma\, \sigma^2_{\epsilon_0}\, \sigma^2_{z_0}}{M + \gamma^2\, \sigma^2_{\epsilon_0}\, \sigma^2_{z_0}}\, P_0 = z_0$$

or, equivalently, if

$$P_0 = \frac{M\left(M + \gamma^2\, \sigma^2_{\epsilon_0}\, \sigma^2_{z_0}\right)}{M^2\left(1 + \sigma^2_{\epsilon_0}\right) + \gamma^2\left(M + \sigma^2_{\epsilon_0}\right)\sigma^2_{\epsilon_0}\, \sigma^2_{z_0}}\, \left(S_0 - \frac{\gamma\, \sigma^2_{\epsilon_0}}{M}\, z_0\right).$$

[36]The second derivative with respect to y_0 is $-\gamma^2\, G_U$, which is clearly negative.

This expression is identical to the price function P_0 defined in Theorem 1.

Similarly, the market-clearing condition at date 1 is given by

$$M\,x_1 + (1 - M)\,y_1 = z_1,$$

or, in terms of total stock holdings, by

$$M\,(x_0 + x_1) + (1 - M)\,(y_0 + y_1) = z_0 + z_1. \tag{28}$$

From (3), (9), and (10), it follows that an informed investor's optimal stock holdings are given by

$$x_0 + x_1 = \frac{1}{\gamma}\left(\frac{1}{\sigma^2_{\epsilon_0}}\,S_0 + \frac{1}{\sigma^2_{\epsilon_1}}\,S_1 - \left(1 + \frac{1}{\sigma^2_{\epsilon_0}} + \frac{1}{\sigma^2_{\epsilon_1}}\right)\,P_1\right). \tag{29}$$

The optimal date 1 stock holdings of an uninformed investor are given by (5). Substituting the equilibrium price coefficients *a, b, c, d, e,* and *f* into the expressions for the conditional expectation and variance of V given by (11) and (12), we have

$$\begin{aligned} y_0 + y_1 \quad &= \quad \frac{\gamma M^2\big((1+\sigma^2_{\epsilon_0})\,\sigma^2_{\epsilon_1}\,\sigma^2_{z_1} - \sigma^2_{\epsilon_0}\,\sigma^2_{z_0}\big) + \gamma^3\,\sigma^4_{\epsilon_0}\,\sigma^2_{\epsilon_1}\,\sigma^2_{z_0}\,\sigma^2_{z_1}}{\big(M^2 + \gamma^2\,\sigma^2_{\epsilon_0}\,\sigma^2_{z_0}\big)\,\big(M + \gamma^2\,\sigma^2_{\epsilon_1}\,\sigma^2_{z_1}\big)\,\sigma^2_{\epsilon_0}}\,\big(P_0 - P_1\big) \\ &\quad - \frac{\gamma\,\sigma^2_{\epsilon_0}\,\sigma^2_{z_0}}{M + \gamma^2\,\sigma^2_{\epsilon_0}\,\sigma^2_{z_0}}\,P_0 \end{aligned} \tag{30}$$

Substituting (29) and (30) into (28), replacing P_0 by $a\,S_0 + b\,z_0$, and solving for P_1, we get

$$\begin{aligned} P_1 \quad &= \quad \frac{M\,\sigma^2_{\epsilon_1}}{D_1}\,(M + \gamma^2\,\sigma^2_{\epsilon_0}\,\sigma^2_{z_0})(M^2 + \gamma^2\,\sigma^2_{\epsilon_1}\,\sigma^2_{z_1})\,\left(S_0 - \frac{\gamma\,\sigma^2_{\epsilon_0}}{M}\,z_0\right) \\ &\quad + \frac{M\,\sigma^2_{\epsilon_0}}{D_1}\,(M^2 + \gamma^2\,\sigma^2_{\epsilon_0}\,\sigma^2_{z_0})(M + \gamma^2\,\sigma^2_{\epsilon_1}\,\sigma^2_{z_1})\,\left(S_1 - \frac{\gamma\,\sigma^2_{\epsilon_1}}{M}\,z_1\right), \end{aligned}$$

where D_1 is defined in Theorem 1. This proves that the prices specified in Theorem 1 clear the market. □

Proof of Proposition 1

In this proof, some (tedious) algebraic steps are only sketched and some complex expressions are omitted for brevity. The details are available from the author upon request.

Suppose the uninformed investors' optimal date 1 stock holdings, which were shown to be linear in P_0 and P_1 in Section 3.2, are given by $y_0 + y_1 = \beta_0 P_0 + \beta_1 P_1$. Further, let $\phi = b/a$ and consider the following linear transformations of the equilibrium prices:

$$\begin{aligned}
\theta_0 &\equiv \frac{1}{a} P_0, \\
\theta_1 &\equiv \frac{\gamma \tau_I}{M \lambda_0} \left(\left(\frac{M}{\gamma \tau_I} - (1-M) \beta_1 \right) P_1 - (1-M) \beta_0 P_0 \right) - \frac{1}{a} P_0.
\end{aligned}$$

Then, $\theta_0 = S_0 + \phi z_0$. Moreover, from the market clearing condition

$$\frac{M}{\gamma \tau_I} (\lambda_0 S_0 + \lambda_1 S_1 - P_1) + (1-M)(\beta_0 P_0 + \beta_1 P_1) = z_0 + z_1,$$

we have

$$\begin{aligned}
\theta_1 &= S_0 + \frac{\lambda_1}{\lambda_0} S_1 - \frac{\gamma \tau_I}{M \lambda_0} (z_0 + z_1) - (S_0 + \phi z_0) \\
&= \frac{\sigma_{\epsilon_0}^2}{\sigma_{\epsilon_1}^2} S_1 - \left(\frac{\gamma \sigma_{\epsilon_0}^2}{M} + \phi \right) z_0 - \frac{\gamma \sigma_{\epsilon_0}^2}{M} z_1.
\end{aligned}$$

Note that θ_0 and θ_1 are informationally equivalent to P_0 and P_1. Thus, the uninformed investors' demand function can be written as

$$y_0 + y_1 = \frac{E[V \mid \theta_0, \theta_1] - P_1}{\gamma Var[V \mid \theta_0, \theta_1]}.$$

Applying the projection theorem, we get

$$y_0 + y_1 = \frac{\hat{\kappa}_0 \theta_0 + \hat{\kappa}_1 \theta_1 - P_1}{\gamma \hat{\tau}_U},$$

where the coefficients $\hat{\kappa}_0$, $\hat{\kappa}_1$, and $\hat{\tau}_U$ are functions of γ, M, σ_{ϵ_0}, σ_{ϵ_1}, σ_{z_0}, σ_{z_1}, and ϕ. Substituting this demand function and the demand function of informed investors given by (29) into the market-clearing condition and solving for P_1 yields

$$P_1 = c S_0 + d S_1 + e z_0 + f z_1,$$

where the coefficients c, d, e, and f depend on γ, M, σ_{ϵ_0}, σ_{ϵ_1}, σ_{z_0}, σ_{z_1}, and ϕ. These price coefficients can now be used to express the informed investors' optimal date 0 demand function given by (4) in terms of S_0, z_0, P_0, the model primitives γ, M, σ_{ϵ_0}, σ_{ϵ_1}, σ_{z_0}, σ_{z_1}, and the price coefficient ϕ:

$$x_0 = \alpha_S\, S_0 + \alpha_z\, z_0 + \alpha_P\, P_0,$$

where both α_S and α_z are rational functions of ϕ with quartic numerator polynomials and identical denominator polynomials. Let $y_0 = \psi\, P_0$ denote the uninformed investors' date 0 demand, where ψ is a function of ϕ. Then the date 0 market-clearing condition, which is given by

$$M\,(\alpha_S\, S_0 + \alpha_z\, z_0 + \alpha_P\, P_0) + (1-M)\,\psi\, P_0 = z_0,$$

implies that

$$\begin{aligned} a &= -\frac{M\,\alpha_S}{M\,\alpha_P + (1-M)\,\psi}, \\ b &= -\frac{M\,\alpha_z - 1}{M\,\alpha_P + (1-M)\,\psi}. \end{aligned}$$

Thus, the price coefficients a, b, c, d, e, and f constitute an equilibrium, if and only if ϕ satisfies the quintic equation

$$\phi = \frac{M\,\alpha_z - 1}{M\,\alpha_S}.$$

Fortunately, this equation can be written as the product of a linear term and two quadratic terms in ϕ:

$$\left(\gamma\,\sigma_{\epsilon_0}^2 + M\,\phi\right)\left(g_0 + g_1\,\phi + g_2\,\phi^2\right)\left(h_0 + h_1\,\phi + h_2\,\phi^2\right) = 0,$$

where

$$\begin{aligned} g_0 &= \sigma_{\epsilon_0}^4\left(M + \gamma^2\,\sigma_{\epsilon_1}^2(\sigma_{z_0}^2 + \sigma_{z_1}^2)\right), \\ g_1 &= \gamma(1+M)\,\sigma_{\epsilon_0}^2\,\sigma_{\epsilon_1}^2\,\sigma_{z_0}^2, \\ g_2 &= \sigma_{z_0}^2\left(M(\sigma_{\epsilon_0}^2 + \sigma_{\epsilon_1}^2) + \gamma^2\,\sigma_{\epsilon_0}^2\,\sigma_{\epsilon_1}^2\,\sigma_{z_1}^2\right), \\ h_0 &= \sigma_{\epsilon_0}^4\Big(M^3(\sigma_{\epsilon_0}^2 + \sigma_{\epsilon_1}^2 + \sigma_{\epsilon_0}^2\,\sigma_{\epsilon_1}^2) + \gamma^4\,\sigma_{\epsilon_0}^2\,\sigma_{\epsilon_1}^4\,\sigma_{z_1}^2(\sigma_{z_0}^2 + \sigma_{z_1}^2) \\ &\quad + \gamma^2\,M\,\sigma_{\epsilon_1}^2\left(\sigma_{\epsilon_0}^2\,\sigma_{z_1}^2 + (M\,\sigma_{\epsilon_0}^2 + (1+\sigma_{\epsilon_0}^2)\,\sigma_{\epsilon_1}^2)(\sigma_{z_0}^2 + \sigma_{z_1}^2)\right)\Big), \\ h_1 &= \gamma\,\sigma_{\epsilon_0}^2\,\sigma_{\epsilon_1}^2\,\sigma_{z_0}^2\left(2\,M^2(\sigma_{\epsilon_0}^2 + \sigma_{\epsilon_1}^2 + \sigma_{\epsilon_0}^2\,\sigma_{\epsilon_1}^2) + \gamma^2(1+M)\,\sigma_{\epsilon_0}^2\,\sigma_{\epsilon_1}^2\,\sigma_{z_1}^2\right), \\ h_2 &= \sigma_{z_0}^2\left(M^2(\sigma_{\epsilon_0}^2 + \sigma_{\epsilon_1}^2) + \gamma^2\,\sigma_{\epsilon_0}^2\,\sigma_{\epsilon_1}^2\,\sigma_{z_1}^2\right) \\ &\quad \times\left(M(\sigma_{\epsilon_0}^2 + \sigma_{\epsilon_1}^2 + \sigma_{\epsilon_0}^2\,\sigma_{\epsilon_1}^2) + \gamma^2\,\sigma_{\epsilon_0}^2\,\sigma_{\epsilon_1}^2\,\sigma_{z_1}^2\right). \end{aligned}$$

This shows that $\phi = -\gamma\, \sigma_{\epsilon_0}^2 / M$ is indeed an equilibrium. Moreover, if the inequality $\sigma_{\epsilon_1} \leq 2\sqrt{M}\, \sigma_{\epsilon_0}/(1-M)$ holds, then $g_1^2 < 4\, g_0\, g_2$ and, hence, $g_0 + g_1\, \phi + g_2\, \phi^2 > 0$ for all $\phi \in \mathbb{R}$. Similarly, if $M \geq \frac{2}{\sqrt{3}} - 1$, then $h_1^2 < 4\, h_0\, h_2$ and $h_0 + h_1\, \phi + h_2\, \phi^2 = 0$ does not have a real root. This proves that if both conditions are satisfied, the equilibrium specified in Theorem 1 is the unique linear REE. □

Proof of Proposition 2

The proof consists of two parts. First, we show that y_1 and ΔP_1 are perfectly positively correlated, if $\sigma_{\epsilon_1}\, \sigma_{z_1} < \hat{\sigma}_1$. Then, we demonstrate that this implies that the probability of an informed investor selling a winner (loser) is greater (less) than $\frac{1}{2}$.

From (27) and (30), it follows immediately that an uninformed investor's stock demand at date 1 can be written as

$$y_1 = -\frac{\gamma M^2 \Big(\big(1 + \sigma_{\epsilon_0}^2\big)\, \sigma_{\epsilon_1}^2\, \sigma_{z_1}^2 - \sigma_{\epsilon_0}^2\, \sigma_{z_0}^2 \Big) + \gamma^3\, \sigma_{\epsilon_0}^4\, \sigma_{\epsilon_1}^2\, \sigma_{z_0}^2\, \sigma_{z_1}^2}{\big(M^2 + \gamma^2\, \sigma_{\epsilon_0}^2\, \sigma_{z_0}^2\big) \big(M + \gamma^2\, \sigma_{\epsilon_1}^2\, \sigma_{z_1}^2\big)\, \sigma_{\epsilon_0}^2}\, \Delta P_1.$$

Thus, y_1 is linearly increasing (decreasing) in ΔP_1, if and only if $\sigma_{\epsilon_1}\, \sigma_{z_1}$ is less (greater) than

$$\hat{\sigma}_1 = \frac{M\, \sigma_{\epsilon_0} \sigma_{z_0}}{\sqrt{M^2 (1 + \sigma_{\epsilon_0}^2) + \gamma^2 \sigma_{\epsilon_0}^4 \sigma_{z_0}^2}}.$$

Next, we show that this linear relationship between y_1 and ΔP_1 puts a lower bound on the probability that an informed investor sells a winning stock. Using the market-clearing condition, we have

$$\begin{aligned} Pr\big(x_1 < 0 \mid x_0 > 0, \Delta P_1 > 0\big) &= Pr\big(z_1 < (1-M)\, y_1 \mid x_0 > 0, \Delta P_1 > 0\big) \\ &> Pr\big(z_1 < 0 \mid x_0 > 0, \Delta P_1 > 0\big), \end{aligned}$$

where the inequality follows from the fact that a price increase implies an increase in the uninformed investors' stock holdings (i.e., $y_1 > 0$), if $\sigma_{\epsilon_1}\, \sigma_{z_1} < \hat{\sigma}_1$. Since x_0, ΔP_1, and z_1 are jointly normally distributed with zero means, Lemma 4 allows us to rewrite the conditional probability as

$$Pr\big(z_1 < 0 \mid x_0 > 0, \Delta P_1 > 0\big)$$

$$= \frac{2\pi - \arccos\rho_{x_0,\Delta P_1} - \arccos(-\rho_{x_0,z_1}) - \arccos(-\rho_{z_1,\Delta P_1})}{2\,(\pi - \arccos\rho_{x_0,\Delta P_1})}$$
$$= \frac{1}{2} - \frac{\arcsin\rho_{z_1,\Delta P_1}}{\pi + 2\arcsin\rho_{x_0,\Delta P_1}},$$

where $\rho_{x_0,\Delta P_1}$ denotes the correlation between x_0 and ΔP_1, $\rho_{z_1,\Delta P_1}$ the correlation between z_1 and ΔP_1, and ρ_{x_0,z_1} the correlation between x_0 and z_1. The second equality follows from the fact that x_0 and z_1 are uncorrelated and that $\arcsin\rho + \arccos\rho = \frac{\pi}{2}$. Thus, the probability of an informed investor selling a winner exceeds $\frac{1}{2}$, if $\arcsin\rho_{z_1,\Delta P_1} < 0$ or, equivalently, if $Cov[z_1, \Delta P_1] = f\,\sigma^2_{z_1} < 0$. This inequality obviously holds, since a positive supply shock increases the required risk premium and thus reduces the date 1 price (i.e., the coefficient f is negative).

Similarly, the conditional probability that an informed investor sells a losing stock can be shown to be less than $\frac{1}{2}$.

$$\begin{aligned} Pr\big(x_1 < 0 \mid x_0 > 0, \Delta P_1 < 0\big) &= Pr\big(z_1 < (1-M)\,y_1 \mid x_0 > 0, \Delta P_1 < 0\big) \\ &< Pr\big(z_1 < 0 \mid x_0 > 0, \Delta P_1 < 0\big) \\ &= \frac{1}{2} + \frac{\arcsin\rho_{z_1,\Delta P_1}}{\pi - 2\arcsin\rho_{x_0,\Delta P_1}} \end{aligned}$$

Thus, the probability is bounded above by $\frac{1}{2}$, since $Cov[z_1, \Delta P_1] < 0$. This proves that informed investors are more likely to sell their winning stocks than their losing stocks, if $\sigma_{\epsilon_1}\,\sigma_{z_1} < \hat{\sigma}_1$. □

Proof of Corollary 1

As shown in the proof of Proposition 2, y_1 is linearly increasing in ΔP_1, if $\sigma_{\epsilon_1}\,\sigma_{z_1} < \hat{\sigma}_1$. Thus, $Cov[y_1, \Delta P_1] > 0$. Furthermore,

$$\begin{aligned} Cov[x_1, \Delta P_1] &= \frac{1}{M}\,Cov[z_1, \Delta P_1] - \frac{1-M}{M}\,Cov[y_1, \Delta P_1] \\ &= \frac{1}{M}\,f\,\sigma^2_{z_1} - \frac{1-M}{M}\,Cov[y_1, \Delta P_1] \\ &< 0, \end{aligned}$$

since f is negative. □

Proof of Proposition 3

Suppose $\sigma_{\epsilon_1}\,\sigma_{z_1} < \hat{\sigma}_1$. Then y_1 is linearly increasing in ΔP_1 (see proof of Proposition 2) and, hence, the condition $\Delta P_1 > 0$ is equivalent to $y_1 > 0$ or, using the market-clearing condition, to $M\,x_1 < z_1$. Thus,

$$\begin{aligned} E\big[x_1 \mid x_0 > 0, \Delta P_1 > 0, x_1 < 0\big] &= E\big[x_1 \mid x_0 > 0, x_1 < \frac{1}{M}\,z_1, x_1 < 0\big] \\ &< E\big[x_1 \mid x_0 > 0, x_1 < 0\big] \\ &< E\big[x_1 \mid x_0 > 0, x_1 > \frac{1}{M}\,z_1, x_1 < 0\big] \\ &= E\big[x_1 \mid x_0 > 0, \Delta P_1 < 0, x_1 < 0\big]. \end{aligned}$$

This proves that the expected trade size of sold winners is larger than that of sold losers. □

Proof of Proposition 4

Let $\gamma = 1$, $M = \frac{1}{2}$, $\sigma_{\epsilon_0} = \sigma_{\epsilon_1} = \sigma_{z_0} = 1$, and $\sigma_{z_1} = 0$. Then y_1 and ΔP_1 are perfectly positively correlated (see proof of Proposition 2), and since there is no date 1 supply shock, x_1 and ΔP_1 are perfectly negatively correlated. Furthermore, x_0, x_1, and ΔP_2 are jointly normally distributed with mean zero and covariance matrix

$$\begin{pmatrix} \sigma^2_{x_0} & \rho_{x_0,x_1}\sigma_{x_0}\sigma_{x_1} & \sigma_{x_0,\Delta P_2} \\ \rho_{x_0,x_1}\sigma_{x_0}\sigma_{x_1} & \sigma^2_{x_1} & \sigma_{x_1,\Delta P_2} \\ \sigma_{x_0,\Delta P_2} & \sigma_{x_1,\Delta P_2} & \sigma^2_{\Delta P_2} \end{pmatrix} = \begin{pmatrix} \frac{19}{8} & \frac{19}{104} & \frac{19}{26} \\ \frac{19}{104} & \frac{67}{1352} & -\frac{15}{338} \\ \frac{19}{26} & -\frac{15}{338} & \frac{95}{169} \end{pmatrix}$$

Thus, from the projection theorem and Lemmas 4 and 5, we have

$$\begin{aligned} &E\big[\Delta P_2 \mid x_0 > 0, \Delta P_1 > 0, x_1 < 0\big] \\ &\quad= E\big[\Delta P_2 \mid x_0 > 0, x_1 < 0\big] \\ &\quad= E\left[\tfrac{1}{3}\,(x_0 + x_1) \mid x_0 > 0, x_1 < 0\right] \\ &\quad= \left(\frac{3}{2\pi}\big(\pi - \arccos(-\rho_{x_0,x_1})\big)\right)^{-1} \frac{1 - \rho_{x_0,x_1}}{2\sqrt{2\pi}}\,(\sigma_{x_0} - \sigma_{x_1}) \\ &\quad= \frac{2\left(67\sqrt{19\pi} + 15\sqrt{67\pi}\right)}{1742\left(\pi - \arccos\left(\sqrt{\frac{19}{67}}\right)\right)} \\ &\quad\approx 0.396 \end{aligned}$$

Similarly,

$$
\begin{aligned}
E\big[\Delta P_2 \mid x_0 > 0, \Delta P_1 < 0, x_1 > 0\big] & \\
&= E\left[\tfrac{1}{3}\,(x_0 + x_1) \mid x_0 > 0, x_1 > 0\right] \\
&= \left(\frac{3}{2\pi}\big(\pi - \arccos \rho_{x_0,x_1}\big)\right)^{-1} \frac{1 + \rho_{x_0,x_1}}{2\sqrt{2\pi}}\,(\sigma_{x_0} + \sigma_{x_1}) \\
&= \frac{67\sqrt{19\pi} - 15\sqrt{67\pi}}{871 \arccos\left(\sqrt{\frac{19}{67}}\right)} \\
&\approx 0.341
\end{aligned}
$$

Thus, for the given parameter values, the expected period 2 price change of winners informed investors sell exceeds that of losers they buy. We want to emphasize that the parameter set under which the result obtains is not a set of measure zero. That is, the result continues to hold for a range of values around the chosen value for each parameter. □

Proof of Proposition 5

First, note that the conditional variance of the period 2 price change can be decomposed as follows:

$$
Var\big[\Delta P_2 \mid \cdot\big] = E\big[Var[\Delta P_2 \mid x_0, \Delta P_1, x_1]\big|\cdot\big] + Var\big[E[\Delta P_2 \mid x_0, \Delta P_1, x_1]\big|\cdot\big],
$$

where the first term on the right-hand side is identical for winners that are sold and for losers that are bought, since x_0, x_1, ΔP_1, and ΔP_2 are jointly normally distributed and the conditional variance $Var[\Delta P_2 \mid x_0, \Delta P_1, x_1]$ does therefore not depend on x_0, x_1, and ΔP_1. Further, since the informed investors' optimal date 1 stock holdings, $x_0 + x_1$, are linearly increasing in $E[V - P_1 \mid \mathcal{F}_1^I]$, where $\mathcal{F}_1^I$ contains x_0 and ΔP_1, we have

$$
\begin{aligned}
E\big[\Delta P_2 \mid x_0, \Delta P_1, x_1\big] &= E\big[V - P_1 \mid x_0 + x_1, \Delta P_1, x_1\big] \\
&= E\big[V - P_1 \mid x_0 + x_1\big] \\
&= \alpha\,(x_0 + x_1),
\end{aligned}
$$

for some positive constant α. Moreover, as σ_{z_1} goes to zero, y_1 is linearly increasing in ΔP_1 and $x_1 = -(1 - M)/M\, y_1$, which implies that

$\lim_{\sigma_{z_1}\downarrow 0} Corr[x_1, \Delta P_1] = -1$. Thus, in order to prove Proposition 5, it suffices to show that

$$Var\big[x_0 + x_1 \mid x_0 > 0, x_1 < 0\big] > Var\big[x_0 + x_1 \mid x_0 > 0, x_1 > 0\big].$$

By Corollary 3, this inequality holds if x_0 and x_1 are negatively correlated, or, since $\lim_{\sigma_{z_1}\downarrow 0} Corr[x_1, \Delta P_1] = -1$, if x_0 and ΔP_1 are positively correlated. Using again the fact that $x_0 + x_1$ is proportional to $E[\Delta P_2 \mid \mathcal{F}_1^I]$, we have

$$Cov[x_0 + x_1, \Delta P_1] = \alpha^{-1} E\big[E[\Delta P_2 \mid \mathcal{F}_1^I]\, \Delta P_1\big] = \alpha^{-1} Cov[\Delta P_1, \Delta P_2],$$

where $\alpha > 0$. Since $\lim_{\sigma_{z_1}\downarrow 0} Cov[\Delta P_1, \Delta P_2] > 0$ (see Proposition 8), this implies that $\lim_{\sigma_{z_1}\downarrow 0} Cov[x_0 + x_1, \Delta P_1] > 0$, and since

$$\lim_{\sigma_{z_1}\downarrow 0} Cov[x_0, \Delta P_1] = \lim_{\sigma_{z_1}\downarrow 0} Cov[x_0 + x_1, \Delta P_1] - \lim_{\sigma_{z_1}\downarrow 0} Cov[x_1, \Delta P_1],$$

that $\lim_{\sigma_{z_1}\downarrow 0} Cov[x_0, \Delta P_1] > 0$, because $\lim_{\sigma_{z_1}\downarrow 0} Cov[x_1, \Delta P_1] < 0$ by Corollary 1. □

Proof of Proposition 6

As shown in the proof of Proposition 2, y_1 and ΔP_1 are perfectly negatively correlated, if $\sigma_{\epsilon_1}\, \sigma_{z_1} > \hat{\sigma}_1$. This implies that

$$Pr\big(y_0 > 0, \Delta P_1 < 0, y_1 < 0\big) = 0$$

and, hence, that uninformed investors are more likely to sell their winning stocks than their losing stocks. If, on the other hand, $\sigma_{\epsilon_1}\, \sigma_{z_1} < \hat{\sigma}_1$, y_1 and ΔP_1 are perfectly positively correlated and

$$Pr\big(y_0 > 0, \Delta P_1 > 0, y_1 < 0\big) = 0.$$

Thus, in this case, uninformed investors are more likely to sell their losers. This proves that the inequality $\sigma_{\epsilon_1}\, \sigma_{z_1} > \hat{\sigma}_1$ is a necessary and sufficient condition for the result that uninformed investors prefer to sell winning stocks. □

Proof of Corollary 2

The negative correlation between y_1 and ΔP_1 follows immediately from the fact that y_1 is linearly decreasing in ΔP_1, if $\sigma_{\epsilon_1}\, \sigma_{z_1} > \hat{\sigma}_1$ (see proof of Proposition 2). □

Proof of Proposition 7

First, we prove the expected-returns inequality. Recall that y_0, y_1, and ΔP_2 are jointly normally distributed and that the uninformed investors' optimal date 1 stock holdings, $y_0 + y_1$, are linearly increasing in $E[V - P_1 \mid \mathcal{F}_1^U]$, where $\mathcal{F}_1^U$ contains y_0 and ΔP_1. Thus,

$$\begin{aligned} E\big[\Delta P_2 \mid y_0, \Delta P_1, y_1\big] &= E\big[V - P_1 \mid y_0 + y_1, \Delta P_1, y_1\big] \\ &= E\big[V - P_1 \mid y_0 + y_1\big] \\ &= \alpha\,(y_0 + y_1). \end{aligned}$$

for some positive constant α.

Kamat (1953) has shown that incomplete moments of trivariate normally distributed random variables are continuous functions of the respective variances and covariances. Thus, the conditional expectations $E[y_0 + y_1 \mid y_0 > 0, \Delta P_1 > 0, y_1 < 0]$ and $E[y_0 + y_1 \mid y_0 > 0, \Delta P_1 < 0, y_1 > 0]$ are continuous in σ_{z_1}. In order to prove the first part of Proposition 7, it therefore suffices to show that the expected-returns inequality holds in the limit as σ_{z_1} goes to $\hat{\sigma}_{z_1} \equiv \hat{\sigma}_1/\sigma_{\epsilon_1}$. From the proof of Proposition 2, we know that y_1 converges to zero in this case. Hence,

$$\lim_{\sigma_{z_1}\downarrow\hat{\sigma}_{z_1}} E\big[y_0 + y_1 \mid y_0 > 0, \Delta P_1 \underset{(<)}{>} 0, y_1 \underset{(>)}{<} 0\big] = \lim_{\sigma_{z_1}\downarrow\hat{\sigma}_{z_1}} E\big[y_0 \mid y_0 > 0, \Delta P_1 \underset{(<)}{>} 0\big].$$

We are therefore left to show that

$$\lim_{\sigma_{z_1}\downarrow\hat{\sigma}_{z_1}} E\big[y_0 \mid y_0 > 0, \Delta P_1 > 0\big] > \lim_{\sigma_{z_1}\downarrow\hat{\sigma}_{z_1}} E\big[y_0 \mid y_0 > 0, \Delta P_1 < 0\big].$$

From Lemmas 4 and 5, it follows immediately that a sufficient condition for this inequality to hold is that $\lim_{\sigma_{z_1}\downarrow\hat{\sigma}_{z_1}} Cov[y_0, \Delta P_1] > 0$. In fact, it is straightforward to show that y_0 and ΔP_1 are positively correlated for all $\hat{\sigma}_{z_1} \le \sigma_{z_1} < \bar{\sigma}_1/\sigma_{\epsilon_1}$. Using again the fact that $y_0 + y_1$ is linearly increasing in $E[\Delta P_2 \mid \mathcal{F}_1^U]$, we have

$$Cov[y_0 + y_1, \Delta P_1] = \beta\, E\big[E[\Delta P_2 \mid \mathcal{F}_1^U]\,\Delta P_1\big] = \beta\, Cov[\Delta P_1, \Delta P_2],$$

for some positive constant β. Since $Cov[\Delta P_1, \Delta P_2] > 0$ for all $\sigma_{z_1} < \bar{\sigma}_1/\sigma_{\epsilon_1}$ (see Proposition 8), this implies that $Cov[y_0 + y_1, \Delta P_1] > 0$, and since

$$Cov[y_0, \Delta P_1] = Cov[y_0 + y_1, \Delta P_1] - Cov[y_1, \Delta P_1],$$

that $Cov[y_0, \Delta P_1] > 0$ for all $\hat{\sigma}_{z_1} \leq \sigma_{z_1} < \bar{\sigma}_1/\sigma_{\epsilon_1}$, because for these parameter values, y_1 is linearly decreasing in ΔP_1 (see proof of Proposition 2). This proves the existence of a $\sigma_1^* > \hat{\sigma}_1$ such that for all $\sigma_{\epsilon_1}\sigma_{z_1} \in (\hat{\sigma}_1, \sigma_1^*)$, the expected period 2 return of winning stocks uninformed investors sell is higher than that of losing stocks they buy.

The proof of the second part of Proposition 7 is analogous to the proof of Proposition 5. The conditional variance of the period 2 price change can again be decomposed into two parts:

$$Var\big[\Delta P_2 \mid \cdot\big] = E\big[Var[\Delta P_2 \mid y_0, \Delta P_1, y_1]\big| \cdot\big] + Var\big[E[\Delta P_2 \mid y_0, \Delta P_1, y_1]\big| \cdot\big],$$

where the first term on the right-hand side is identical for winners that are sold and for losers that are bought, since y_0, y_1, ΔP_1, and ΔP_2 are jointly normally distributed. Further, since $E[\Delta P_2 \mid y_0, \Delta P_1, y_1] = E[\Delta P_2 \mid y_0 + y_1]$, and since y_1 is linearly decreasing in ΔP_1 for $\sigma_{\epsilon_1}\sigma_{z_1} > \hat{\sigma}_1$, it suffices to show that

$$Var\big[y_0 + y_1 \mid y_0 > 0, y_1 < 0\big] > Var\big[y_0 + y_1 \mid y_0 > 0, y_1 > 0\big].$$

By Corollary 3, this inequality holds if y_0 and y_1 are negatively correlated, or, equivalently, if y_0 and ΔP_1 are positively correlated, which was shown to be true for all $\hat{\sigma}_1 \leq \sigma_{\epsilon_1}\sigma_{z_1} < \bar{\sigma}_1$. □

Proof of Proposition 8

The covariance of ΔP_1 and ΔP_2 is equal to

$$\begin{aligned}
Cov[\Delta P_1, \Delta P_2] &= Cov[(c-a)\, S_0 + d\, S_1 + (e-b)\, z_0 + f\, z_1, \\
&\qquad V - c\, S_0 - d\, S_1 - e\, z_0 - f\, z_1] \\
&= (c-a)\left(1 - c\left(1+\sigma_{\epsilon_0}^2\right) - d\right) + d\left(1 - c - d\left(1+\sigma_{\epsilon_1}^2\right)\right) \\
&\quad - e(e-b)\, \sigma_{z_0}^2 - f^2\, \sigma_{z_1}^2 \\
&= K\Big(M^2\left(\sigma_{\epsilon_0}^2\, \sigma_{z_0}^2 - \left(1+\sigma_{\epsilon_0}^2\right)\sigma_{\epsilon_1}^2\, \sigma_{z_1}^2\right) \\
&\quad + \gamma^2\, \sigma_{\epsilon_0}^2\, \sigma_{z_0}^2\left(M\, \sigma_{\epsilon_0}^2\, \sigma_{z_0}^2 - \left(M+\sigma_{\epsilon_0}^2\right)\sigma_{\epsilon_1}^2\, \sigma_{z_1}^2\right)\Big),
\end{aligned}$$

where K is a strictly positive function of γ, M, σ_{ϵ_0}, σ_{ϵ_1}, σ_{z_0}, and σ_{z_1}. Thus, the price changes are positively correlated, if and only if

$$\sigma_{\epsilon_1}^2\, \sigma_{z_1}^2 < \frac{M\, \sigma_{\epsilon_0}^2\, \sigma_{z_0}^2\left(M + \gamma^2\, \sigma_{\epsilon_0}^2\, \sigma_{z_0}^2\right)}{M^2\left(1+\sigma_{\epsilon_0}^2\right) + \gamma^2\, \sigma_{\epsilon_0}^2\, \sigma_{z_0}^2\left(M+\sigma_{\epsilon_0}^2\right)} = \bar{\sigma}_1^2.$$

□

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