Chapter 4: Voting and Social Choice.

Topics:

- Ordinal Welfarism
- Condorcet and Borda: 2 alternatives for majority voting
- Voting over Resource Allocation
- Single-Peaked Preferences
- Intermediate Preferences
- Preference Aggregation and Arrow's Impossibility Theorem

1. Ordinal Welfarism.

Some situations: cardinal welfarism impossible

Ordinal welfarism: based on *ordering* of individual preferences.

Central Postulate: individual welfare entirely captured by a preference ordering of possible outcomes

A: set of possible outcomes, or choice set

R: relation between options: *complete* and *transitive*

Rational choices: if R exists such that selected S(B) is highest of the ranked outcomes according to R.

'Welfarist Program' (in it's ordinal form) needed to analyze social choice to analyze which compromises are just. Example: Pareto.

Two social choice theory models:	interpretations
• Voting Problem	normative (benevolent dictator) or strategic voting (private info)
• Preference Aggregation Problem	order all outcomes, search for collective preference instead of 'best' Use collective utility function.

2. Condorcet and Borda.

Critique on plurality (majority) voting: only ‘top’ of preferences matters: entire preference relation ignored.

Borda: weight all personal preferences in A; highest score wins
Condorcet: based on majority relation: b P^m a, b P^m c, c P^m a → b wins.

Remarks:

- Borda-scores (scoring methods) are in fact cardinal utilities, unrelated to personal feelings
- Borda and Condorcet can lead to the same, as well as to different results, dependent on the relative position of preferences: Borda takes into account entire preference profile, Condorcet also focuses on whole profile, but not ‘at once’.
- Major problem of this: Condorcet method may lead to cycles
  Possible outcome: delete weakest link: smallest majority ignored.
  Problem then: ‘Reunion Paradox’: total group split in two, one cycles (delete w.l), one not, then unified: not necessarily the same winner as if Condorcet method is applied in whole group.
  Note: Any scoring method (Borda) is immune to this problem.

3. Voting over Resource Allocation.

Elections sometimes not over small group of options, so impractical to address scores to potential outcomes (a large choice set A: how to divide a homogeneous private good among group of selfish people?).

Problem: relatively small coalitions can impose large negative externalities on other groups. Destructive Competition: veto power (for several possible coalitions!) occurs, leading to cycles → instability and unpredictability!

4. Single-Peaked Preferences.

Which characteristics for individual preferences are needed for a transitive majority relation?

If preferences are single-peaked, transitivity of the majority relation is guaranteed. Example: how to locate a facility among the line [0,1]?

Assumptions:

- Large number of voters, spread continuously in [0,1],
- Disutility in case of living far away from it: $u_i = - |y - x_i|$
- Distribution F: at location z: 1 – F(z) living on [z,1] (no one living at z)
- Median of F is y*: F (y*) = ½.

→ y* is the classical utilitarian solution as well as the Condorcet-winner.

Definition: Preference Relation R is single-peaked (in the ordering of A) with peak x^i , if x^i is the top outcome of R_i in A, and for all outcome x ($\neq x^i$), R_i prefers any outcome between x^i and x to x itself.

Majority relation is transitive and single-peaked. Consequence: In case of single-peaked preferences, agents only need to report their peaks, which leads to a Condorcet-winner.

If so: no incentive to lie: report peak is always best strategy (the majority always consists of 'true peaks').

Condorcet-method preferred above all scoring-methods with respect to this problem, because all of these methods fail to be strategy proof (even if preferences are single-peaked).

Conclusion: Condorcet-method useful if the outcomes can be arranged along a one-dimensional line and individual preferences are single peaked.

(As soon as the single-peaked assumption is not fulfilled anymore, also this method fails the strategy proof. Cycling and hence the undesired instability in the voting process occurs. → Search for another assumption about preferences to make sure that the majority relation is transitive.)

5. Intermediate Preferences.

I.P. is a second assumption about preferences such that a Condorcet winner exists, or in other words, that the majority relation in transitive.

I.P. method: ordering of agents instead of outcomes (single peaked pref.)

If agents I, j both aPb → so do all agents in between i and j.

Now the majority relation is transitive if [N(a,b) + N(b,c)] > ½ N;\

That is: the majority has aPb and bPc.

Example 4.7: agents voting over which surplus sharing model will be used.

Example 4.8: even in absence of s.p.p., the IP-property still holds, so that majority voting always delivers a Condorcet-winner.

Conclusion: a majority ranking exists, even in the absence of s.p.p.

6. Preference Aggregation and Arrow's Impossibility Theorem.

Recall: social choice problem consists of the choice set A, N agents and the preference relation R_i (N agents choose *a* according to their R_i).

Solve the problem of different personal preferences if society chooses with an *aggregation method* F: social preference relation R* = F(R).

Assumptions made in Social Choice Theory:

- Process leading to social outcome should be based on well-founded axioms,
- This process should allow positive predictions (no instability).

Unfortunately: Arrow's Impossibility Theorem:

The search for rationality of collective choice is hopeless
(if these assumptions have to be taken in account).

Two simple aggregation methods:

- Condorcet: general will: $R^m = F(R)$
  Problem: some R's cycle, leading to instability. Deleting these weakest links contrasts to first assumption: freedom of choice for everyone.

- Borda provides aggregation method for all preference profiles in A, and the majority relation is transitive.

  Problem in this case: majority may choose a out of (a,b,c), whereas the Borda winner might be b because of its relative position over c. In other words: this means that the overall outcome b is not independent of the irrelevant outcome c
  Because the contest is between a and b, c should be irrelevant according to the assumption of *Independence of Irrelevant Alternatives*:
  > A collective R, R*, should only depend on individual preferences between concerned outcomes.

Suppose Condorcet method is accepted (delete w.l.).
> Even then it's not a correct aggregation method, because IIA violated!

Arrow's Impossibility Theorem
> Any aggregation function leading to a rational collective preference and obeying the IIA-principle, must be very undesirable because of its lack of efficiency or of fairness.

- If efficient aggregation method is needed, the only rational solution becomes one of dictatorship, which contrasts with fairness.
- A fair distribution: if always the same R_0 is selected (very inefficient!)

Two solutions:

- Restriction of domain: exclude non-single peaked or non-intermediate preferences
- Weaken rationality: allow several possible majorities (yet: instability!)