Interest Rate Control and Nonconvergence to Rational Expectations

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This paper investigates the feasibility of a monetary policy aimed at pegging the nominal rate of interest. It shows that under general conditions such a policy would produce the well-known cumulative process, despite the fact that there exists a well-behaved rational expectations equilibrium with no tendency for inflation to accelerate or decelerate. The cumulative process shows up as the failure of learning to converge to rational expectations. Specifically, the paper shows, first in a conventional IS-LM model with an expectations-augmented Phillips curve and then in a micro-based finance constraint model, that if people follow any learning rule based on experience that satisfies a weak condition, then the sequence of temporary equilibria under a policy of interest pegging cannot converge. The nonconvergent path that will be observed accords with the familiar cumulative process, in that inflation accelerates if the market rate of interest has been pegged below the natural rate.

I. Introduction

In his 1968 presidential address to the American Economic Association, Milton Friedman argued, among other things, that controlling interest rates tightly was not a feasible monetary policy. His argument was a variation on Knut Wicksell's cumulative process.[1] Start in full

I wish to thank, without implicating, Olivier Blanchard, Joel Fried, Meir Kohn, David Laidler, Robert Lucas, and two anonymous referees for useful comments on earlier drafts.

[1] As Friedman noted, his account differed from Wicksell's in several important ways, especially in the central role assigned by Friedman to the Fisherian distinction between *real and nominal interest rates. In order to avoid misleading historical connotations,* the text below refers to the kind of instability described by Friedman as "the" cumulative process. For a detailed account of Wicksell's own analysis, see Laidler (1991, chap. 5).

[*Journal of Political Economy,* 1992, vol. 100, no. 4]

employment with no actual or expected inflation. Let the monetary authority peg the nominal interest rate below the natural rate. This will require monetary expansion, which will eventually cause inflation. When expected inflation rises in response to actual inflation, the Fisher effect will put upward pressure on the interest rate. More monetary expansion will be required to maintain the peg. This will make inflation accelerate until the policy is abandoned. Likewise, if the interest rate is pegged above the natural rate, deflation will accelerate until the policy is abandoned. Since no one knows the natural rate, the policy is doomed one way or another.

This argument, which was once quite uncontroversial, at least among monetarists, has lost its currency. One reason is that the argument invokes adaptive expectations, and there appears to be no way of reformulating it under rational expectations. McCallum (1986) shows that in conventional rational expectations models, monetary policy can peg the nominal rate, even in the face of random shifts in the natural rate, without producing runaway inflation or deflation. This can be accomplished by systematically reducing monetary growth and hence reducing expected inflation, whenever the natural rate is expected to rise. Furthermore, as McCallum points out, pegging the nominal rate at a lower value will produce a lower average rate of inflation, not the ever-higher inflation predicted by Friedman.

There is a rational expectations argument, due to Sargent and Wallace (1975), to the effect that interest pegging will make the price level indeterminate. However, as McCallum and others have pointed out, the indeterminacy in the Sargent-Wallace argument arises from a failure to specify monetary policy fully. Furthermore, as Laidler (1983) has emphasized, indeterminacy is not the same as the dynamic instability of the cumulative process.

Thus the rational expectations revolution has almost driven the cumulative process from the literature. Modern textbooks treat it as a relic of pre–rational expectations thought. For example, Sargent's (1987, p. 99) brief reference characterizes Wicksell's stability analysis as merely "important in the history of economic thought." The only other reference in the same text (p. 461) identifies Wicksell's "observation" as indeterminacy of the price level. Blanchard and Fischer (1989, pp. 577–80) present a model of the cumulative process but warn the reader that it is "hard to believe that individuals continue to form expectations of inflation adaptively if the inflation rate is ever-accelerating." They go on to examine interest pegging under rational expectations and conclude that the danger of instability is "not inherent to such a policy." McCallum (1989) does not even refer to the cumulative process.

With the disappearance of Friedman's argument there has been a revival in belief that interest pegging is feasible, desirable, and even conducive to stability. Barro (1989) argues that interest pegging is "reasonable" (p. 4) and presents evidence to the effect that the Federal Reserve Board has in fact pegged interest rates, although randomly, since World War II. Woodford (*1990b*) argues that whereas monetary control may yield rational expectations equilibria with the price level affected by extrinsic (sunspot) uncertainty, interest pegging does not.

The purpose of this paper is to argue that, contrary to these rational expectations arguments, the cumulative process is not only possible but inevitable, not just in a conventional Keynesian macro model but also in a flexible-price, micro-based, finance constraint model, whenever the interest rate is pegged. The paper follows Laidler (1983) and Cottrell (1989) in arguing that the essence of the cumulative process lies not in an economy's rational expectations equilibria but in the disequilibrium adjustment process by which people try to acquire rational expectations. It argues that, under a wide set of assumptions, the process cannot converge if the monetary authority keeps interest rates pegged and that the cumulative process is a manifestation of this nonconvergence.

Thus the cumulative process should be regarded not as a relic but as an implication of real-time belief formation of the sort studied in the literature on convergence (or nonconvergence) to rational expectations equilibrium (e.g., Frydman and Phelps 1983). Several recent authors (e.g., Evans 1985; Marcet and Sargent 1989; Woodford 1990*a*) have suggested using the approach of this literature to address the question of which equilibrium will actually be reached in a world of multiple equilibria. The present paper uses the approach instead to address the question of what kind of monetary policy allows any rational expectations equilibrium at all to be reached in a world in which multiplicity is not at issue.

Section II below lays out a conventional macro model exhibiting the cumulative process under adaptive but not rational expectations, when the nominal interest rate is pegged. Section III, which contains the main result of the paper, shows that the same process will appear under any learning rule that satisfies a minimal assumption, the violation of which would arguably imply that people are refusing to learn from experience. Section IV extends the result to a finance constraint model. Section V considers further generalizations. Section VI shows that the cumulative process will arise under less rigid interest control, and characterizes the extent to which the monetary authority can control the rate of interest without generating the process. Section VII contains concluding remarks and suggestions for further research.

II. A Conventional Model of the Cumulative Process

Friedman's argument is formalized by the following equation system:

$$y_t = y\left(RE_t\left(\frac{1}{\pi_{t+1}}\right)\right), \quad R > 1, y' < 0, \tag{1}$$

$$\pi_{t+1}E_t\left(\frac{1}{\pi_{t+1}}\right) = f(y_t), \quad f, f' > 0, f(y^*) = 1, \tag{2}$$

$$E_t\left(\frac{1}{\pi_{t+1}}\right) - E_{t-1}\left(\frac{1}{\pi_t}\right) = \gamma\left[\frac{1}{\pi_t} - E_{t-1}\left(\frac{1}{\pi_t}\right)\right], \quad 0 < \gamma < 1, \tag{3}$$

where y_t is demand for output, $\pi_{t+1} \equiv P_{t+1}/P_t$ the inflation factor, and R the (pegged) nominal interest factor. The IS curve is (1); (2) is an expectations-augmented Phillips curve, in which the left-hand side is the average price setter's expected relative price $P_{t+1}E_t(1/P_{t+1})$ and y^* is the natural level of output; and (3) is a convenient form of adaptive expectations. Assuming R pegged makes the LM curve redundant. The natural interest factor (one plus the natural interest rate) is r^*, with

$$y(r^*) = y^*, \quad r^* > 0. \tag{4}$$

Define

$$x_t \equiv \frac{1}{\pi_t}, \tag{5}$$

$$\hat{x}_t \equiv E_{t-1}x_t, \tag{6}$$

and

$$x^* \equiv \frac{r^*}{R} > 0, \tag{7}$$

where x_t is the "return to hoarding money." Define the function

$$h(x) \equiv [f(y(Rx))]^{-1}. \tag{8}$$

The IS and Phillips curves imply

$$x_{t+1} = \hat{x}_{t+1}h(\hat{x}_{t+1}), \quad h, h' > 0, h(x^*) = 1. \tag{9}$$

Formally, adaptive expectations can be written as

$$\hat{x}_{t+1} - \hat{x}_t = \gamma(x_t - \hat{x}_t), \quad 0 < \gamma < 1. \tag{10}$$

From (9) and (10),

$$\hat{x}_{t+1} - \hat{x}_t = \gamma\hat{x}_t[h(\hat{x}_t) - 1]. \tag{11}$$

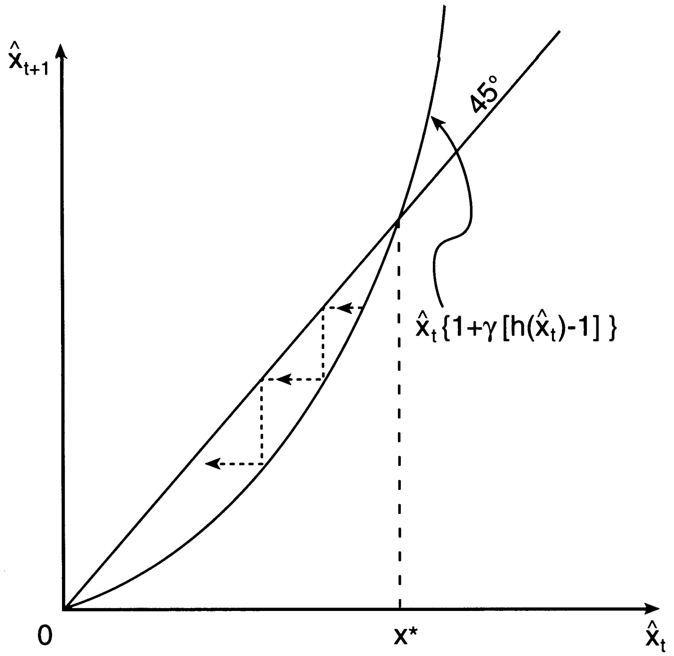

FIG. 1.—The cumulative process under adaptive expectations. The expected return to hoarding money falls to zero and inflation explodes when the market rate is less than the natural rate ($\hat{x}_t < x^*$).

Equation (11) describes the evolution of $\hat{x}_t$ given any initial guess $\hat{x}_0$. It has two rest points: 0 and x^*. The former is a degenerate equilibrium with infinite inflation. The latter is the usual rational expectations equilibrium with a constant, finite rate of inflation that depends positively on the nominal rate of interest: $\pi^* \equiv 1/x^* \equiv R/r^*$. The former rest point is stable and the latter unstable, as shown in figure 1.

The source of the instability of the usual rational expectations equilibrium is the result in (9) that $h' > 0$. Because of this, any departure of the expectation $\hat{x}_{t+1}$ from its equilibrium value x^* will cause an overreaction of the actual value x_{t+1}, which in turn will generate a false signal. Specifically, if people have been overly pessimistic, in the sense that $\hat{x}_{t+1} < x^*$, they will be led to believe instead that they have been overly optimistic, because they will observe $x_{t+1} < \hat{x}_{t+1}$.

To see the connection with Friedman's analysis more clearly, take an initial situation with any given expectation $\hat{x}_0$. Suppose that the monetary authority chooses to peg the nominal rate of interest at such a low level that the actual real rate of interest (the market rate) is below the natural rate. Then in the initial situation people are overly pessimistic. That is, since the market rate is $R\hat{x}_0 - 1$ and the natural rate (by [7]) is $Rx^* - 1$, $\hat{x}_0 < x^*$. Then, as figure 1 makes clear, people will experience even more inflation than they had expected, and that will cause them to be progressively more pessimistic over time. According to (9), as expected inflation explodes, so does actual inflation.

III. The Cumulative Process with General Learning

Adaptive expectations can be thought of as a particular rule by which people attempt to form rational expectations. The cumulative process implies that the rule does not work. Therefore, it is unreasonable to suppose, as above, that the rule will be used forever. People will make fuller use of their adaptive capabilities.

On the other hand, it would not be reasonable to assume people capable of forming rational expectations ab ovo. Even if nature endows us with innate capabilities of various sorts, it seems highly unlikely that the capability of rational macroeconomic forecasting is one of them. Rational expectations can be acquired only through experience, if at all.

These considerations suggest that $\hat{x}_t$ should depend on experience, through a learning rule that allows maximal flexibility but that can also be specified independently of the macroeconomic relationship (9) that generates the variable being forecast. Suppose, accordingly, that

$$\hat{x}_{t+1} = J_t(\Omega_t), \quad t \geq 0, \tag{12}$$

where $\Omega_t \equiv \{x_\tau\}_0^t$ and $\{J_t\}$ is a sequence of functions. The initial guess $\hat{x}_0$ is arbitrary.

One example would be the least-squares learning rule of Bray and Savin (1986) and Marcet and Sargent (1989), which in this case amounts to using the sample mean

$$\hat{x}_{t+1} = (t+1)^{-1} \sum_0^t x_\tau. \tag{13}$$

The rule (13) could also be rationalized as the way a nonparametric econometrician, or a Bayesian with diffuse priors, would forecast x_{t+1} under the maintained hypothesis that $\{x_\tau\}_0^\infty$ is a sequence of independent draws from an unchanging distribution. If the economy were actually in a rational expectations equilibrium, that maintained hypothesis would be correct (with $x_t = x^*$ for all t).

However, a rule as simple as (13) would be no more likely than adaptive expectations to survive if forecasts did not converge to rational expectations. More generality is needed. Accordingly, assume merely that the J_t's pay attention to experience, in the following sense.

ASSUMPTION 1. For any $t \geq 1$, (i) if, for all $\tau = 0, \ldots, t-1$, $x_\tau < \hat{x}_\tau$ and (if $\tau \geq 1$) $x_\tau < x_{\tau-1}$, then $\hat{x}_t < \hat{x}_{t-1}$; (ii) if, for all $\tau = 0, \ldots, t-1$, $x_\tau > \hat{x}_\tau$ and (if $\tau \geq 1$) $x_\tau > x_{\tau-1}$, then $\hat{x}_t > \hat{x}_{t-1}$.

The first part of assumption 1 asserts that if the return to hoarding money has always been overestimated in the past and has always fallen in the past, then this period's forecast will be lower than last period's. The second part asserts the analogous result in the reverse situation. If people violated assumption 1, they would be refusing to learn from experience.

Assumption 1 is satisfied by adaptive expectations and by the least-squares rule (13). Grandmont and Laroque (1986) study the case of a fixed rule with finite memory:

$$\hat{x}_t = J(x_{t-2}, \ldots, x_{t-n}). \tag{14}$$

This is a special case of (12) if it is supplemented by n additional rules $J_0, \ldots, J_{n-1}$ to handle the initial period in which the sample size is too small for (14) to apply. If J has strictly positive first-order partial derivatives, then parts i and ii of assumption 1 will be satisfied for all $t > n$.

The cumulative process will arise under interest pegging if people use any sequence of learning rules satisfying assumption 1. Again, the key to the process is the fact that $h' > 0$, which makes the system overreact to a shortfall of the expected return below x^*. The false signal of an actual return less than expected will lead people to reduce their expectation even further below x^*. Proposition 1 states this more formally.

PROPOSITION 1. Suppose that $\{x_t, \hat{x}_t\}_0^\infty$ are generated by (9) and (12), with $\hat{x}_0 > 0$ given. Let $\{J_t\}_0^\infty$ satisfy assumption 1. If $\hat{x}_0 < x^*$, then $\{\hat{x}_t\}_0^\infty$ is a strictly decreasing sequence. If $\hat{x}_0 > x^*$, then $\{\hat{x}_t\}_0^\infty$ is a strictly increasing sequence.

Proof. Take the case in which $\hat{x}_0 < x^*$. Suppose, contrary to the proposition, that $\{\hat{x}_t\}$ is not strictly decreasing. Let t be the first date at which $\hat{x}_t \geq \hat{x}_{t-1}$. By construction,

$$\hat{x}_{t-1} < \hat{x}_{t-2} < \ldots < \hat{x}_0 < x^*.$$

From this and (9),

$$h(\hat{x}_{t-1}) < h(\hat{x}_{t-2}) < \ldots < h(\hat{x}_0) < 1.$$

From these inequalities and (9),

$$x_\tau = \hat{x}_\tau h(\hat{x}_\tau) < \hat{x}_\tau \quad \forall \tau = 0, 1, \ldots, t-1$$

and

$$x_\tau = \hat{x}_\tau h(\hat{x}_\tau) < \hat{x}_{\tau-1} h(\hat{x}_{\tau-1}) = x_{\tau-1} \quad \forall \tau = 1, 2, \ldots, t-1.$$

This establishes the premise of part i of assumption 1. Therefore, $\hat{x}_t < \hat{x}_{t-1}$, a contradiction. The proof for the case $\hat{x}_0 > x^*$ is analogous. Q.E.D.

IV. A Finance Constraint Model of the Cumulative Process

The present section shows that the price stickiness and lack of microfoundations of the model above are inessential to the cumulative process by constructing a finance constraint model with an almost identical dynamic structure. Consider a world such as that described by Lucas (1980) in which agents are endowed with a flow of perishable commodities that appear identical to the outside observer. All agents have an absolute aversion to consuming their own endowments and trade using fiat money, with a payment lag that induces a finance constraint (Clower 1967; Kohn 1981) on their decisions.

Time is discrete. Each period, trade proceeds as follows. First, each agent receives the money for goods sold in the previous period. Then a bond market opens, in which promises to pay one unit of money with certainty next period are traded for money, and the bonds issued last period are redeemed. There is no lag in receipt of payment for bond sales. Finally, the goods market convenes, in which money available this period is exchanged for goods available this period.

Assume instantaneous price flexibility. An alternative source of nonneutrality is needed in order for the monetary authority to have even a momentary influence on the nominal interest rate when people's expectations are predetermined. To this end, the model has heterogeneous agents, and monetary policy produces a systematic distribution effect.[2] Specifically, there are two types of representative agents. The first, called shortsighted, have a two-period horizon. The second, called farsighted, have an infinite horizon.

All quantities are expressed as a ratio to the number of farsighted agents. Shortsighted agents comprise a sequence of overlapping generations with a constant endowment of e when young and derive

[2] This channel of monetary policy was frequently invoked by classical quantity theorists to explain the short-run nonneutrality of money (Patinkin 1972). It was also used more recently by Grossman and Weiss (1983) and Rotemberg (1984), in whose models monetary injections reduce the rate of interest by redistributing real cash balances toward those individuals who are in the market this period and have a relatively low propensity to spend out of real balances, and away from those who are out, who spend all their cash. Distribution effects also play an important role in overlapping generation models.

utility only from consumption when old. Thus each period the young sell e and the old spend eP_{t-1}, the proceeds from last period's sale.

Farsighted agents have a constant endowment y. They receive lump-sum transfers from the monetary authority at the beginning of each period. They face a finance constraint and a wealth constraint:

$$P_t c_t \leq M_{t-1} + T_t + B_{t-1} - \frac{B_t}{R_{t+1}},$$
$$M_t - M_{t-1} = P_t y + T_t + B_{t-1} - P_t c_t - \frac{B_t}{R_{t+1}}, \tag{15}$$

where c_t is consumption, M_{t-1} is money held just before the transfer T_t is received, and B_t is the number of bonds demanded.

Subject to (15) for all t and a no-Ponzi-game constraint, the farsighted agent maximizes $E_t \sum_{j=0}^{\infty} \beta^j u(c_{t+j})$, where $\beta = 1/(1 + \delta)$, $\delta > 0$, and u is a smooth function on R_+ with $u' > 0$, $u'' < 0$, and $\lim_{c\to 0} u'(c) = \infty$. If $R_{t+1} > 1$, necessary first-order conditions are

$$u'(c_t) = R_{t+1} E_t \left[\frac{\beta u'(c_{t+1})}{\pi_{t+1}} \right] \tag{16}$$

and

$$M_t = P_t y. \tag{17}$$

Condition (16) is the usual condition for life cycle consumption choices. Condition (17) asserts that the finance constraint must be binding.

Each period, the price level and interest rate clear the goods and bond markets. Total consumption will equal total supply, $e + y$, of which the shortsighted consume $eP_{t-1}/P_t = e/\pi_t$. Therefore,

$$c_t = y + e\left(1 - \frac{1}{\pi_t}\right). \tag{18}$$

Inflation imposes a tax on the money holdings of the shortsighted and hence raises c_t in (18). This tax is the forced savings that earlier writers recognized as a channel for the real effects of monetary policy.[3]

If $R_t > 1$, then (17) implies that the total money held in the economy at the start of period t will be $M_{t-1}^S = P_{t-1}(y + e)$. Define $\mu_t \equiv M_t^S/M_{t-1}^S$. It follows that if R_t and $R_{t+1} > 1$, then

$$\pi_t = \mu_t. \tag{19}$$

[3] See Hayek (1932) for references to this notion among early classical writers.

Assume

$$R > \max\left\{1, \frac{e}{\beta(e+y)}\right\}. \tag{20}$$

Then it is straightforward to verify that there exists a unique stationary perfect-foresight equilibrium with the constant interest factor R and that it has the constant inflation factor

$$\pi^* \equiv R\beta. \tag{21}$$

Thus the natural rate of interest, $(R/\pi^*) - 1$, is the rate of time preference δ.

When the economy is not in a rational expectations equilibrium, the beliefs underlying the expectations operator E_t in (16) are complicated to describe in full. But according to (16) there is only one parameter of these beliefs that matters for the choice of c_t, namely, the term multiplying R_{t+1} on the right-hand side of (16). This term plays the same role as $\hat{x}_{t+1}$ in the previous section's model, and it has the same interpretation: the expected return to hoarding money. For purposes of the present model, redefine x_t as

$$x_t \equiv \frac{\beta u'\{y + e[1 - (1/\pi_t)]\}}{\pi_t}. \tag{22}$$

It follows that in order for the monetary authority to keep $R_{t+1} = R$, the monetary expansion factor must equal π_t, which in turn must be a solution to

$$u'\left[y + e\left(1 - \frac{1}{\pi_t}\right)\right] = R\hat{x}_{t+1}. \tag{23}$$

A (positive) solution exists and is unique if and only if

$$\hat{x}_{t+1} > \frac{u'(y+e)}{R} \tag{24}$$

and can be expressed as

$$\pi_t = \pi(\hat{x}_{t+1}) > 0, \tag{25}$$

with

$$\pi' = \frac{R\pi^2}{eu''} < 0. \tag{26}$$

To interpret (26), note that $R\hat{x}_{t+1}$ is the opportunity cost of consumption to the farsighted. If $\hat{x}_{t+1}$ increases, the increased cost will reduce farsighted consumption. To clear the goods market at an unchanged nominal rate of interest, the monetary authority must reduce forced saving, allowing the shortsighted to consume more.

When (24) is not satisfied, the interest peg is no longer feasible. The interpretation is that with such a low opportunity cost the farsighted will want to consume more than the economy's gross national product, so no amount of forced saving will clear the goods market. The rate of interest must rise.

Redefine the function h as

$$h(x) \equiv \frac{\beta R}{\pi(x)}. \tag{27}$$

It follows that[4]

$$x_t = \hat{x}_{t+1} h(\hat{x}_{t+1}), \quad h, h' > 0, h(x^*) = 1, \tag{28}$$

where

$$x^* \equiv \frac{u'\{y + e[1 - (1/R\beta)]\}}{R}. \tag{29}$$

Note that x^* is the value of x_t in the perfect-foresight equilibrium, is well defined, and satisfies the existence condition (24).[5]

Equation (28) plays the role of (9) in the previous model, which contained the key to the cumulative process. The only difference is the timing: x_t appears on the left side of (28) in place of x_{t+1}, reflecting the assumption of price flexibility. Because $h' > 0$, the economy will overreact to any shortfall of the expected future return to money below x^*, by reducing the current realized return even further below x^*.

The economic reason for overreaction in this case closely follows Friedman's argument. Start in perfect-foresight equilibrium with $x_t = \hat{x}_{t+1} = x^*$ and then reduce the nominal rate of interest. According to (29), this will raise x^* without changing $\hat{x}_{t+1}$; people will now be too pessimistic. The fall in the market rate will induce the farsighted to consume more, and this increase in demand will drive up inflation. Both reactions will reduce x_t ($= \beta u'(c_t)/\pi_t$) below $\hat{x}_{t+1}$, even though $\hat{x}_{t+1}$ has been made lower than x^*.

Out of perfect-foresight equilibrium, $\hat{x}_{t+1}$ derives from people's beliefs concerning the distribution of future inflation, interest rates, and transfers and their beliefs concerning how their beliefs will change in the future. It is reasonable to suppose that the individual takes all these things as given. But there is one determinant of an

[4] From (23), $u'(y + e\{1 - [1/\pi(x^*)]\}) = Rx^*$. From this and (29), $\pi(x^*) = R\beta$. Substituting this into (27) yields $h(x^*) = 1$.

[5] The right-hand side of (29) is well defined if $y + e[1 - (1/R\beta)] > 0$, which follows from (20); it satisfies (24) because $u'' < 0$ and, by (20), $1/R\beta > 0$.

individual's return on money that he or she chooses, namely own current consumption. An increase in c_t will result in less wealth at $t + 1$ and hence a different (generally higher) value of $u'(c_{t+1})$. To simplify the analysis, assume that the individual ignores the effect of his own current consumption when predicting the return to money. (This assumption is relaxed at no substantive cost in the Appendix.) He treats the problem of determining $\hat{x}_{t+1}$ as one of forecasting an exogenous random variable.

Instability of x^* under learning is what one would expect from the results of Grandmont and Laroque (1986). By substituting x_{t+1} for $\hat{x}_{t+1}$ in (28), we see that x^* is stable under perfect-foresight dynamics or "indeterminate." Grandmont and Laroque show that indeterminacy in a one-dimensional system like (28) implies local instability under the rule (14) with the additional assumption that the rule can "detect" cycles of order two.[6] Furthermore, proposition 2 of Evans (1985) implies that x^* would exhibit "expectational instability"[7] in a linearized version of the model in which the forecast of x_{t+1} is a function of x_{t-1}. It also follows from the connection established by Marcet and Sargent (1989, n. 8) between "expectational stability" and stability of least-squares learning that x^* would be unstable in a linearized model if the forecast of x_{t+1} was a rolling ordinary least squares regression on x_{t-1}. The following argument demonstrates instability under a broad class of learning rules, using an argument almost identical to that of proposition 1.

To avoid simultaneous interactions between x_t and $\hat{x}_{t+1}$, assume that the information available at t does not include x_t. That is, $\Omega_t \equiv \{x_\tau\}_0^{t-1}$. As before,

$$\hat{x}_{t+1} = J_t(\Omega_t), \quad t = 1, 2, \ldots, \tag{30}$$

and the initial guess $\hat{x}_1$ is arbitrary.

Assumption 1 must now be supplemented to include the initial revision at date 1 (when $\hat{x}_2$ is formed). At that date, people have not yet observed a forecast error. The amended assumption follows.

ASSUMPTION 1′. (*a*) (i) If $x_0 < \hat{x}_1$, then $\hat{x}_2 \le \hat{x}_1$; (ii) if $x_0 > \hat{x}_1$, then $\hat{x}_2 \ge \hat{x}_1$. (*b*) For any $t \ge 2$, (i) if, for all $\tau = 1, \ldots, t-1$, $x_\tau < \hat{x}_\tau$ and $x_\tau \le x_{\tau-1}$, then $\hat{x}_{t+1} < \hat{x}_t$; (ii) if, for all $\tau = 1, \ldots, t-1$, $x_\tau > \hat{x}_\tau$ and $x_\tau \ge x_{\tau-1}$, then $\hat{x}_{t+1} > \hat{x}_t$.

The new part *a* simply asserts that if a forecast is revised before any forecast error is observed, it will be revised upward if the first

[6] This follows immediately from theorem 1.1 of Grandmont and Laroque in the case of $k = 1$.

[7] That is, the constant forecast $\hat{x}_{t+1} = x^*$ is not the limit of the sequence in which, at each stage, the forecast function is the equilibrium of the model under the assumption that everyone's expectations are given by the forecast function of the previous stage, except when the forecast in the initial stage is identically x^*.

observation (for date 0) is greater than the first forecast (for date 1) or downward in the opposite case. The inequality in part *b* is weakened to allow the possibility that no revision is made at date 1.

Given assumption 1′, the cumulative process will again be inevitable. As before, if $\hat{x} < x^*$, the economy will overreact; subsequent forecasts will all be revised downward and will never catch up with the fall in x_t. During this process the market rate, $[R/\pi(\hat{x}_{t+1})] - 1$, will be less than the natural rate, $[R/\pi(x^*)] - 1 = \beta^{-1} - 1 = \delta$, and by (26), inflation will be rising. Proposition 1′ states this more formally.

PROPOSITION 1′. Suppose that $\{x_t, \hat{x}_{t+1}\}_0^\infty$ is generated by (28) and (30) with $\hat{x}_1 > 0$ given. Let $\{J_t\}_1^\infty$ satisfy assumption 1′. If $\hat{x}_1 < x^*$, then $\hat{x}_2 \leq \hat{x}_1$ and $\{\hat{x}_t\}_2^\infty$ is a strictly decreasing sequence. If $\hat{x}_1 > x^*$, then $\hat{x}_2 \geq \hat{x}_1$ and $\{x_t\}_2^\infty$ is a strictly increasing sequence.

Proof. Take the case in which $\hat{x}_1 < x^*$. By (28), $x_0 = \hat{x}_1 h(\hat{x}_1) < \hat{x}_1 h(x^*) = \hat{x}_1$. So by part *a*.i of assumption 1′,

$$\hat{x}_2 \leq \hat{x}_1. \tag{31}$$

So it remains to show that $\{\hat{x}_t\}_2^\infty$ is strictly decreasing. Suppose the contrary. Let t be the first date ≥ 2 such that $\hat{x}_{t+1} \geq \hat{x}_t$. Then by (31) and the definition of t,

$$\hat{x}_t < \ldots < \hat{x}_2 \leq \hat{x}_1 < x^*.$$

From this and (28),

$$h(\hat{x}_t) < \ldots < h(\hat{x}_2) \leq h(\hat{x}_1) < 1.$$

From these inequalities and (28),

$$x_\tau = \hat{x}_{\tau+1} h(\hat{x}_{\tau+1}) < \hat{x}_{\tau+1} \leq \hat{x}_\tau, \quad \forall \tau = 1, \ldots, t-1,$$

and

$$x_\tau = \hat{x}_{\tau+1} h(\hat{x}_{\tau+1}) \leq \hat{x}_\tau h(\hat{x}_\tau) = x_{\tau-1}, \quad \forall \tau = 1, \ldots, t-1,$$

This establishes the premise of part *b*.i of assumption 1′. Therefore, $\hat{x}_{t+1} < \hat{x}_t$, a contradiction. The proof for the case $\hat{x}_1 > x^*$ is analogous. Q.E.D.

V. Generalizing the Analysis

The analysis can be generalized in several directions without losing the cumulative process. For example, heterogeneity of preferences[8]

[8] Except for the rate of time preference δ, which we assume the same for all agents in order to avoid the well-known degeneracy of long-run equilibrium with additive intertemporal preferences and heterogeneous rates of time preference.

and beliefs among farsighted agents can be accommodated. Let there be n farsighted agents, with utility functions u_i, learning rules J_{it}, and initial guesses $\hat{x}_{i1}$, $i = 1, \ldots, n$. Then

$$x_{it} \equiv \frac{\beta u_i'(c_{it})}{\pi_t}, \quad i = 1, \ldots, n, \tag{22'}$$

$$u_i'(c_{it}) = R\hat{x}_{it+1}, \quad i = 1, \ldots, n, \tag{23'}$$

and

$$\sum_{i=1}^{n} c_{it} = y + e\left(1 - \frac{1}{\pi_t}\right). \tag{18'}$$

If a temporary equilibrium exists at all with the interest factor pegged at R, then π_t will be the solution to (23′) and (18′):

$$\pi_t = \pi(\hat{x}_{1t+1}, \ldots, \hat{x}_{nt+1}) > 0, \tag{25'}$$

with

$$\pi \text{ decreasing in } \hat{x}_{it+1}, \quad i = 1, \ldots, n. \tag{26'}$$

Redefine h as

$$h(x_1, \ldots, x_n) \equiv \frac{\beta R}{\pi(x_1, \ldots, x_n)}. \tag{27'}$$

Then

$$x_{it} = \hat{x}_{it+1} h(\hat{x}_{1t+1}, \ldots, \hat{x}_{nt+1}), \quad i = 1, \ldots, n; t = 0, 1, \ldots, \qquad h > 0, h \text{ increasing in each argument.} \tag{28'}$$

In a stationary rational expectations equilibrium, $h(x_1, \ldots, x_n) = 1$. Suppose

$$\hat{x}_{it+1} = J_{it}(\Omega_{it}), \quad i = 1, \ldots, n; t = 1, 2, \ldots, \tag{30'}$$

where $\Omega_{it} \equiv \{x_{i\tau}\}_0^{t-1}$. Interpret assumption 1′ as applying to each of the rules $\{J_{it}\}$. Define $\hat{x}_t \equiv (\hat{x}_{1t}, \ldots, \hat{x}_{nt})$. Then the following proposition holds.

Proposition 1″. Suppose that $\{x_{1t}, \ldots, x_{nt}; \hat{x}_{1t+1}, \ldots, \hat{x}_{nt+1}\}_0^\infty$ is generated by (28′) and (30′) with $\hat{x}_1 > 0$ given. Let $\{J_{it}\}_1^\infty$ satisfy assumption 1′ for all i. If $h(\hat{x}_1) < 1$, then $h(\hat{x}_2) \leq h(\hat{x}_1)$ and $\{h(\hat{x}_t)\}_2^\infty$ is a strictly decreasing sequence. If $h(\hat{x}_1) > 1$, then $h(\hat{x}_2) \geq h(\hat{x}_1)$ and $\{h(\hat{x}_t)\}_2^\infty$ is a strictly increasing sequence. Unless $h(\hat{x}_1) = 1$, inflation diverges from the rational expectations equilibrium value $\pi^* = R\beta$.

Proof. Take the case in which $h(\hat{x}_1) < 1$. By (28′), $x_{i0} = \hat{x}_{i1} h(\hat{x}_1) <$

$\hat{x}_{i1}$ for all i. So by part a.i of assumption 1′,

$$\hat{x}_{i2} \leq \hat{x}_{i1} \quad \forall\, i. \tag{31′}$$

By (28′) and (31′), $h(\hat{x}_2) \leq h(\hat{x}_1)$. To show that $\{h(\hat{x}_t)\}_2^\infty$ is strictly decreasing, by (28′) it suffices to show that $\{\hat{x}_{it}\}_2^\infty$ is strictly decreasing for all i. Suppose the contrary. Let t be the first date ≥ 2 such that, for some j, $\hat{x}_{jt+1} \geq \hat{x}_{jt}$. By (31′) and the definition of t,

$$\hat{x}_{it} < \ldots < \hat{x}_{i2} \leq \hat{x}_{i1} \quad \forall\, i.$$

From these and (28′),

$$h(\hat{x}_t) < \ldots < h(\hat{x}_2) \leq h(\hat{x}_1) < 1.$$

From these inequalities and (28′),

$$x_{i\tau} = \hat{x}_{i\tau+1} h(\hat{x}_{\tau+1}) < \hat{x}_{i\tau+1} \leq \hat{x}_{i\tau} \quad \forall\, i, \forall\, \tau = 1, \ldots, t-1,$$

and

$$x_{i\tau} = \hat{x}_{i\tau+1} h(\hat{x}_{\tau+1}) \leq \hat{x}_{i\tau} h(\hat{x}_\tau) = x_{i\tau-1} \quad \forall\, i, \forall\, \tau = 1, \ldots, t-1.$$

This establishes the premises of part b.i of assumption 1′ for each i. Therefore, $\hat{x}_{it+1} < \hat{x}_{it}$ for all i, a contradiction. Since $h(\hat{x}_{t+1})$ starts below 1 and falls forever, it follows from (25′) and (27′) that π_t starts above π^* and rises forever. The proof for the case $h(\hat{x}_1) > 1$ is analogous. Q.E.D.

Likewise, the analysis can be generalized to allow intrinsic uncertainty. This generalization is important not because intrinsic uncertainty is needed in order to have a meaningful distinction between interest pegging and monetary control (variations in expectations during the learning process are sufficient for this purpose) but because (a) the cumulative process is robust to adding random variables on which people could condition their beliefs and (b) the instability proofs above rely on a strict monotonicity of $\{x_t\}$ that might not occur with intrinsic randomness.

The model of Section III can easily incorporate random shocks to the natural rate. Replace the IS curve (1) with

$$y_t = y(R\hat{x}_{t+1}, \theta_t), \quad y_1 < 0, \tag{1′}$$

where θ_t is independently and identically distributed on the finite set $\{\theta^1, \ldots, \theta^n\}$. There is a separate natural rate $r^{*i} > 0$ for each θ^i, satisfying $y(r^{*i}, \theta^i) = y^*$. Define

$$x^{*i} \equiv \frac{r^{*i}}{R} > 0 \quad \forall\, i \tag{7′}$$

and

$$h(x, \theta) \equiv [f(y(Rx, \theta))]^{-1}. \tag{8′}$$

The IS and Phillips curves yield

$$x_{t+1} = \hat{x}_{t+1} h(\hat{x}_{t+1}, \theta_t), \quad h, h_1 > 0, h(x^{*i}, \theta^i) = 1 \; \forall \, i. \tag{9'}$$

People can observe the realization of θ_t when forecasting x_{t+1}. Suppose that they follow the practice of nonparametric econometricians with enough data and base their forecast exclusively on the experience that has been associated with the same realization. Define

$$S_t^i \equiv \{\tau | \tau \leq t \text{ and } \theta_{\tau-1} = \theta^i\}.$$

Whenever $\theta_t = \theta^i$, they use a learning rule $J_t^i(\Omega_t^i)$, where $\Omega_t^i \equiv \{x_\tau | \tau \in S_t^i\}$. Then the evolution of the economy over all dates $t + 1$ with $\theta_t = \theta^i$ will proceed independently of what happens at any other date and will be given by (9′) and

$$\hat{x}_{t+1} = J_t^i(\Omega_t^i). \tag{12'}$$

Assumption 1 must be modified to restrict all inequalities to $\tau \in S_t^i$. The analysis is then formally equivalent to that of Section III, and the cumulative process again emerges.

Note that the same argument would imply a cumulative process if the monetary authority attempted to peg the rate of interest at different values depending on the state: at $R^i - 1$ whenever $\theta_{t-1} = \theta^i$, with not all R^i's equal. This shows that what generates instability is the attempt to stabilize interest rates across time, not across states. (By the same token, a policy that kept the rate of monetary expansion following a state-contingent path that would yield a rational expectations equilibrium with a constant nominal rate of interest and a stationary distribution of inflation might not generate the cumulative process, because it would not be attempting to stabilize interest rates across time, except in the long run when people's beliefs have converged to rational expectations.)

In general, however, it is not possible for the adjustment processes associated with different states to proceed independently of one another. When they do not, the analysis of dynamics becomes complicated, and results under very general learning rules are hard to obtain. However, the cumulative process will still be exhibited under more special rules.

An extreme example of interdependence arises when people can observe x_t only with observational error. They cannot condition expectations on this random error because it is unobservable. Suppose, for example, in the finance constraint model, that the individual observes $x_t + \theta_t$ instead of the true return, where θ_t is independently and identically distributed with bounded support and $E(\theta_t) = 0$. Suppose that people use the sample mean of these noisy observations as their estimator:

$$\hat{x}_{t+1} = \frac{1}{t}\sum_{0}^{t-1}(x_\tau + \theta_\tau).$$

Equivalently,

$$\hat{x}_{t+1} - \hat{x}_t = \frac{1}{t}(x_{t-1} - \hat{x}_t + \theta_{t-1}).$$

From this and (28),

$$\hat{x}_{t+1} - \hat{x}_t = \frac{1}{t}\{\hat{x}_t[h(\hat{x}_t) - 1] + \theta_{t-1}\}. \tag{32}$$

The analysis of Ljung (1977) can now be applied. Consider the differential equation whose right-hand side is the expected value of the term multiplying $1/t$ on the right-hand side of (32):

$$\frac{d}{dt}\hat{x} = \hat{x}[h(R\hat{x}) - 1]. \tag{33}$$

The two stationary points of (33) are 0 and x^*. By theorem 2 of Ljung,[9] $\hat{x}_t$ cannot converge to x^* with positive probability if the derivative of the right-hand side of (33) is positive at x^*. That derivative is $Rx^*h'(x^*) > 0$. Therefore, $\hat{x}_t$ cannot converge with positive probability to x^*.

Of course the cumulative process is not perfectly general: it is possible to construct models in which learning converges under interest pegging. However, such generalizations tend to yield peculiar implications. In the case of the conventional macro model in Section III, an upward-sloping IS curve ($y' > 0$) or a Phillips curve that showed price setters reducing prices in the face of excess demand ($f' < 0$) could avoid the cumulative process. For then $h' < 0$ in (9). However, either of these features would appear implausible from a conventional Keynesian or monetarist perspective.

Likewise, the finance constraint model of Section IV could be modified to allow stability under interest pegging. Suppose, for example, that all the transfers went to the old shortsighted agents instead of going to farsighted agents. Then a simple accounting exercise shows that as long as the nominal rate of interest was always positive, con-

[9] It is easily verified that the system (32) satisfies assumptions A of Ljung. The other two premises of the theorem ([24] and [25] of Ljung) are satisfied if the variance of θ_t is positive.

sumption of the farsighted would equal $c_t = y/\pi_t$, so that

$$x_t = \frac{\beta u'(y/\pi_t)}{\pi_t} \tag{22''}$$

and

$$u'\left(\frac{y}{\pi_t}\right) = R_{t+1}\hat{x}_{t+1}. \tag{23''}$$

In this case, setting $R_{t+1} = R$ and solving (23″) would yield $\pi(\hat{x}_{t+1})$ with $\pi' > 0$. An increase in the expected return to holding money would raise the price level, holding the interest rate pegged. This is the analogue to an upward-sloping IS curve. Instability would not necessarily arise under an interest peg because $h' < 0$ in (28).

Not only would there be an analogue to an upward-sloping IS curve, but the impact effect of monetary policy on interest rates would also be perverse. If the authority kept μ_t on a preset course and allowed R_{t+1} to be determined by (23″), then, as before, $\mu_t = \pi_t$ if the nominal rate of interest was always positive. So from (23″), with the expectation $\hat{x}_{t+1}$ held fixed, an increase in the rate of monetary expansion μ_t ($= \pi_t$) would raise the nominal (and real) interest factor ($\partial R_{t+1}/\partial \mu_t = -U''y/\hat{x}_{t+1}\pi_t^2 > 0$). In this case, by giving transfers to the shortsighted, the monetary authority would be causing forced dissaving.[10]

Thus within the class of models considered above, generalizations that would not yield the cumulative process would also yield subsidiary implications at odds with standard macroeconomic analysis. At the least this should create a presumption that instability will be the consequence of interest pegging.

VI. Alternative Monetary Policies

The cumulative process is not inevitable under a pegged rate of monetary expansion. In the finance constraint model, if μ_t were held constant at μ, then as long as R_{t+1} remained greater than unity π_t

[10] The fact that in this modified model the positive effect of monetary expansion on nominal interest rates also implies a positive effect of inflation on interest rates (since $\pi_t = \mu_t$) does not make the model more plausible relative to the model of Sec. IV, in which inflation has a negative effect on interest rates, because in either case the effect works when expected inflation is held constant, whereas the standard interpretation of the positive empirical effect of inflation on nominal interest rates requires expected inflation to change.

would equal μ, by (19). Thus x_t would equal the constant

$$\frac{\beta u'\{y + e[1 - (1/\mu)]\}}{\mu}.$$

Under any reasonable learning rule, $\hat{x}_{t+1}$ would converge to this constant.[11]

This section examines the related question of whether a less rigid form of interest control might avoid the cumulative process. Specifically, consider the finance constraint model and suppose that the monetary authority allows the rate of interest to adjust to inflation, according to

$$R_{t+1} = a + b\pi_t, \quad b > 0. \tag{34}$$

The pure peg studied above is the limiting case with $a > 0$ and $b = 0$. When $a > 0$, the rule (34) allows the "backward-looking real rate" $(R_{t+1}/\pi_t) - 1$ to fall when inflation rises, but not by as large a proportion as under a pure peg. When $a < 0$, the rule makes the nominal rate "overreact" to inflation by raising the backward-looking real rate.

The case of $a = 0$, where the authority attempts to keep the real rate pegged, is the dividing line between stability and instability. If the nominal rate does not overreact to inflation, then the cumulative process will arise. Otherwise beliefs can converge.

In perfect-foresight equilibrium the inflation and interest factors must satisfy $\pi^* = R^*\beta$ as before. So from (34),

$$\pi^* = \frac{a\beta}{1 - b\beta}, \quad R^* = \frac{a}{1 - b\beta}. \tag{35}$$

In order for this equilibrium to be well defined, assume the equivalent of (20):

$$\frac{a}{1 - b\beta} > \max\left\{1, \frac{e}{\beta(e + y)}\right\}. \tag{36}$$

From (23) and (34),

$$u'\left[y + e\left(1 - \frac{1}{\pi_t}\right)\right] = (a + b\pi_t)\hat{x}_{t+1}. \tag{37}$$

[11] This result relies heavily on the fixed velocity of circulation in the finance constraint model, which rules out Cagan's (1956) stability problem. However, Marcet and Sargent (1989, pp. 356–57) examine the Cagan problem and show convergence for certain parameter values in a linear model under least-squares learning. Likewise, Evans (1985, p. 1229) shows expectational stability of the fundamental equilibrium in a variable-velocity model with a fixed money supply, for certain parameter values.

From (36) and the implicit function theorem, a unique (positive) solution $\pi(\hat{x}_{t+1})$ exists to (37), with $a + b\pi(\hat{x}_{t+1}) > 1$, in a neighborhood of $x^* \equiv \beta u'\{y + e[1 - (1/\pi^*)]\}/\pi^* > 0$, with[12]

$$\pi(x^*) \equiv \pi^* \tag{38}$$

and

$$\pi'(x) < 0. \tag{39}$$

So for $\hat{x}_{t+1}$ in this neighborhood,

$$\pi_t = \pi(\hat{x}_{t+1}) > 0. \tag{40}$$

Redefine h once more as

$$h(x) \equiv \frac{\beta a}{\pi(x)} + \beta b. \tag{41}$$

Then in a neighborhood of x^*,

$$h > 0, \quad h' \gtrless 0 \text{ as } a \gtrless 0, \tag{42}$$

and

$$h(x^*) = 1. \tag{43}$$

From (22), (37), (40), and (41),

$$x_t = \hat{x}_{t+1} h(\hat{x}_{t+1}). \tag{44}$$

In the case in which $a > 0$, (42)–(44) imply (28). So under assumption 1′, by proposition 1′, the cumulative process will arise. However, when $a < 0$, convergence can occur. Consider, for example, the least-squares learning rule:

$$\hat{x}_{t+1} = \frac{1}{t}\sum_{0}^{t-1} x_\tau. \tag{45}$$

Then

$$\hat{x}_{t+1} - \hat{x}_t = \frac{1}{t}\hat{x}_t[h(\hat{x}_t) - 1]. \tag{46}$$

Since $h' < 0$, the system (46) is easily seen to converge to x^* if it remains in the neighborhood in which h is well defined. From (46),

[12] From (36), x^* is well defined. Let $f(\pi, x) \equiv u'\{y + e[1 - (1/\pi)]\} - (a + b\pi)x$. Then $f(\pi^*, x^*) = (x^*\pi^*/\beta) - (a + b\pi^*)x^*$, by the definition of x^*; this and the definition of π^* imply $f(\pi^*, x^*) = 0$. Note that $f_1(\pi, x) < 0$ when $f(\pi, x)$ is well defined and $(\pi, x) > 0$. So π^* is the unique solution to (37) when $x = x^*$. Also, by (36), $a + b\pi^* > 1$. The extension of $\pi(x)$ in a neighborhood of x^* follows from the implicit function theorem, with $\pi'(x) = (a + b\pi)/f_1 < 0$.

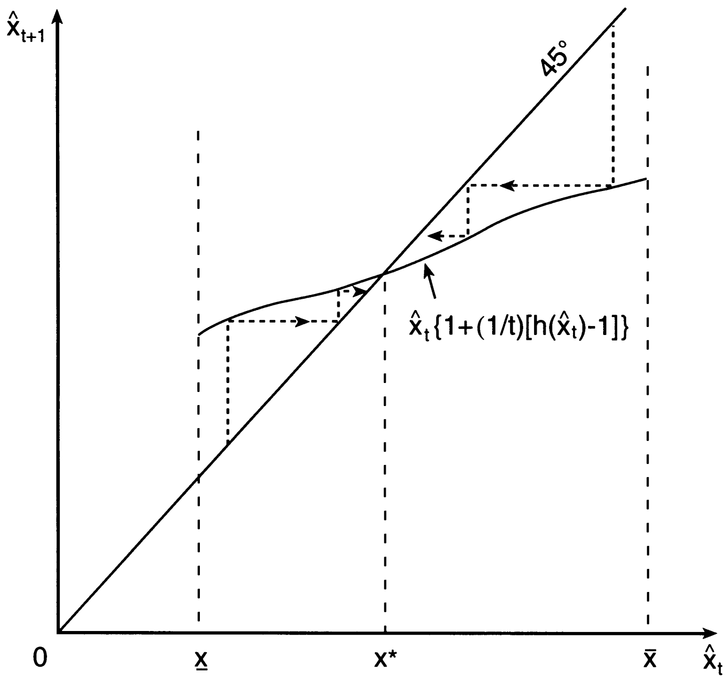

FIG. 2.—When the real rate of interest is made an increasing function of the rate of inflation, convergence can occur.

$0 < \partial\hat{x}_{t+1}/\partial\hat{x}_t < 1$ for large t. So for large t, $\hat{x}_t$ will not leave the neighborhood $(\underline{x}, \bar{x})$ in which h is well defined and $a + b\pi > 1$ (see fig. 2).

VII. Conclusion

In summary, the analysis above shows a critical difference between pegging the rate of monetary expansion and pegging the rate of interest: stability is possible under the former but not under the latter.[13] Although well-behaved rational expectations equilibria exist, they are not stable when the monetary authority attempts to dampen the effect of changing expectations on the nominal rate of interest. The instability that follows from interest stabilization is a manifesta-

[13] Boyd and Dotsey (1990) investigate the expectational stability (as in Evans [1985]) of various equilibria under interest smoothing policies. Their results are not strictly comparable to those of the present paper since they assume that the central bank maintains an unchanged money supply rule during the adjustment process, whereas the current paper assumes that it is the interest rate rule that is kept unchanged.

tion of the cumulative process. This process is an inevitable consequence of interest pegging in both a conventional Keynesian macro model and a flexible-price, finance constraint model, and under minimal assumptions on learning behavior. Under the same assumptions on learning, it is also an inevitable consequence of a more flexible interest control policy, as long as the monetary authority tries to prevent the rate of interest from adjusting as much as point for point with the rate of inflation.

It should be emphasized that the dynamic structure of the model above is extremely simple. This source of instability might conceivably be offset by the interaction between learning and other forms of adjustment in a more complicated multidimensional system. For example, a low-interest policy might offset the cumulative process by stimulating investment and hence driving the natural rate down to the market rate.[14] It would be interesting to explore such issues in a more general setting.

It should also be emphasized that the analysis assumes that all information Ω_t available for use in conditioning the forecasts $\hat{x}_{t+1}$ consists of observations generated under the regime of interest pegging. This raises the question of whether experience under earlier, different regimes might be employed to make expectations converge.

The long-run nature of our results should also be stressed. The analysis of Poole (1970) suggests that interest stabilization might constitute a sensible short-run policy if there is enough randomness in the demand-for-money function. The present analysis is best taken as a warning against extending that kind of policy too far into the future.

Perhaps the most important lesson of the analysis is that the assumption of rational expectations can be misleading, even when used to analyze the consequences of a fixed monetary regime. If the regime is not conducive to expectational stability, then the consequences can be quite different from those predicted under rational expectations. An economist who focused only on rational expectations equilibria would conclude from the models above that a policy of interest control would produce stable inflation in the long run; whereas in fact, if our analysis is correct, it would lead to accelerating inflation or deflation and, possibly, to the collapse of the regime.

This suggests that, in general, any rational expectations analysis of monetary policy should be supplemented with a stability analysis of the sort conducted in this paper, to determine whether or not the rational expectations equilibrium could ever be observed. It also sug-

[14] Wicksell ([1905] 1935, pp. 198–99) explicitly recognized that the forced saving associated with low interest rates might have this effect.

gests expectational stability as a criterion for evaluating alternative monetary policies.

Appendix

This Appendix generalizes the analysis of Section IV to allow $\hat{x}_{t+1}$ to depend (positively) on c_t. Define

$$\Omega'_t \equiv \{x_0, \ldots, x_{t-1}; c_0, \ldots, c_t\} \equiv (\Omega''_t, c_t), \quad t = 1, 2, \ldots,$$

and let

$$\hat{x}_{t+1} = J_t(\Omega'_t), \quad t = 1, 2, \ldots, \tag{30''}$$

where the functions J_t satisfy the following assumption.

ASSUMPTION A1′. (*a*) (i) If $x_0 < \hat{x}_1$ and $c_1 \leq c_0$, then $\hat{x}_2 \leq \hat{x}_1$; (ii) if $x_0 > \hat{x}_1$ and $c_1 \geq c_0$, then $\hat{x}_2 \geq \hat{x}_1$. (*b*) For any $t \geq 2$, (i) if $c_t \leq c_{t-1}$ and, for all $\tau = 1, \ldots, t-1$, $x_\tau < \hat{x}_\tau$ and $x_\tau \leq x_{\tau-1}$, then $\hat{x}_{t+1} < \hat{x}_t$; (ii) if $c_t \geq c_{t-1}$ and, for all $\tau = 1, \ldots, t-1$, $x_\tau > \hat{x}_\tau$ and $x_\tau \geq x_{\tau-1}$, then $\hat{x}_{t+1} > \hat{x}_t$. (*c*) J_t is nondecreasing in c_t, with Ω''_t held fixed, for all $t \geq 1$.

This modification of assumption 1′ allows the individual to anticipate that higher consumption today may reduce wealth and thus increase the marginal utility of consumption next period. It also allows him to increase the forecast $\hat{x}_{t+1}$ despite a history of overestimated and falling x_τ, if he is also planning to increase c_t. Nevertheless, the cumulative process will still arise. From (18), (25), and (26),

$$c_t = c(\hat{x}_{t+1}) \equiv y + e\left[1 - \frac{1}{\pi(\hat{x}_{t+1})}\right], \quad c' < 0, t = 0, 1, \ldots. \tag{A1}$$

Thus the following proposition holds.

PROPOSITION A1′. Suppose that $\{c_t, x_t, \hat{x}_{t+1}\}_0^\infty$ is generated by (28), (30″), and (A1), with $\hat{x}_1 > 0$ given. Let $\{J_t\}_1^\infty$ satisfy assumption A1′. If $\hat{x}_1 < x^*$, then $\hat{x}_2 \leq \hat{x}_1$ and $\{\hat{x}_t\}_2^\infty$ is a strictly decreasing sequence. If $\hat{x}_1 > x^*$, then $\hat{x}_2 \geq \hat{x}_1$ and $\{\hat{x}_t\}_2^\infty$ is a strictly increasing sequence.

Proof. Take the case in which $\hat{x}_1 < x^*$. By (28), $x_0 = \hat{x}_1 h(\hat{x}_1) < \hat{x}_1 h(x^*) = \hat{x}_1$. By this and part *a*.i of assumption A1′, $J_1(\Omega''_1, c_0) \leq \hat{x}_1$. Suppose, contrary to the proposition, that $\hat{x}_2 > \hat{x}_1$. Then, by (A1), $c_1 < c_0$. By this and part *c* of assumption A1′,

$$\hat{x}_2 = J_1(\Omega''_1, c_1) \leq J_1(\Omega''_1, c_0).$$

Therefore, $\hat{x}_2 \leq \hat{x}_1$, a contradiction. This establishes that

$$\hat{x}_2 \leq \hat{x}_1. \tag{31''}$$

So it remains to show that $\{\hat{x}_t\}_2^\infty$ is strictly decreasing. Suppose the contrary. Let t be the first date ≥ 2 such that $\hat{x}_{t+1} \geq \hat{x}_t$. Note that, by (31″) and the definition of t,

$$\hat{x}_t < \ldots < \hat{x}_2 \leq \hat{x}_1 < x^*.$$

From this and (28),

$$h(\hat{x}_t) < \ldots < h(\hat{x}_2) \leq h(\hat{x}_1) < 1.$$

From these inequalities,

$$x_\tau = \hat{x}_{\tau+1} h(\hat{x}_{\tau+1}) < \hat{x}_{\tau+1} \leq \hat{x}_\tau, \quad \tau = 1, \ldots, t-1,$$

and

$$x_\tau = \hat{x}_{\tau+1} h(\hat{x}_{\tau+1}) \leq \hat{x}_\tau h(\hat{x}_\tau) = x_{\tau-1}, \quad \tau = 1, \ldots, t-1.$$

It follows from part *b*.i of assumption A1′ that

$$J_t(\Omega''_t, c_{t-1}) < \hat{x}_t.$$

By (A1),

$$c_t = c(\hat{x}_{t+1}) \leq c(\hat{x}_t) = c_{t-1}.$$

So by part *c* of assumption A1′,

$$\hat{x}_{t+1} = J_t(\Omega''_t, c_t) \leq J_t(\Omega''_t, c_{t-1}).$$

Therefore, $\hat{x}_{t+1} < \hat{x}_t$, a contradiction. The proof for the case $\hat{x}_1 > x^*$ is analogous. Q.E.D.

References

Barro, Robert J. "Interest-Rate Targeting." *J. Monetary Econ.* 23 (January 1989): 3–30.

Blanchard, Olivier J., and Fischer, Stanley. *Lectures on Macroeconomics.* Cambridge, Mass.: MIT Press, 1989.

Boyd, John H., and Dotsey, Michael. "Interest Rate Rules and Nominal Determinacy." Working Paper no. 222. Rochester, N.Y.: Univ. Rochester, Center Econ. Res., February 1990.

Bray, Margaret M., and Savin, Nathan E. "Rational Expectations Equilibria, Learning, and Model Specification." *Econometrica* 54 (September 1986): 1129–60.

Cagan, Phillip. "The Monetary Dynamics of Hyperinflation." In *Studies in the Quantity Theory of Money,* edited by Milton Friedman. Chicago: Univ. Chicago Press, 1956.

Clower, Robert W. "A Reconsideration of the Microfoundations of Monetary Theory." *Western Econ. J.* 6 (December 1967): 1–8.

Cottrell, Allin. "Price Expectations and Equilibrium When the Interest Rate Is Pegged." *Scottish J. Polit. Econ.* 36 (May 1989): 125–40.

Evans, George. "Expectational Stability and the Multiple Equilibria Problem in Linear Rational Expectations Models." *Q.J.E.* 100 (November 1985): 1217–33.

Friedman, Milton. "The Role of Monetary Policy." *A.E.R.* 58 (March 1968): 1–17.

Frydman, Roman, and Phelps, Edmund S., eds. *Individual Forecasting and Aggregate Outcomes: "Rational Expectations" Examined.* New York: Cambridge Univ. Press, 1983.

Grandmont, Jean-Michel, and Laroque, Guy. "Stability of Cycles and Expectations." *J. Econ. Theory* 40 (October 1986): 138–51.

Grossman, Sanford J., and Weiss, Laurence. "A Transactions Based Model of the Monetary Transmission Mechanism." *A.E.R.* 73 (December 1983): 871–80.

Hayek, Friedrich A. von. "A Note on the Development of the Doctrine of 'Forced Saving.'" *Q.J.E.* 47 (November 1932): 123–33.

Kohn, Meir. "In Defense of the Finance Constraint." *Econ. Inquiry* 19 (April 1981): 177–95.

Laidler, David. "Misconceptions about the Real Bills Doctrine and the Quantity Theory: A Comment on Sargent and Wallace." Research Report no. 8314. London: Univ. Western Ontario, Dept. Econ., May 1983.

———. *The Golden Age of the Quantity Theory*. Hemel Hempstead: Allan, 1991.

Ljung, Lennart. "Analysis of Recursive Stochastic Algorithms." *IEEE Trans. Automatic Control* 22 (August 1977): 551–75.

Lucas, Robert E., Jr. "Equilibrium in a Pure Currency Economy." *Econ. Inquiry* 18 (April 1980): 203–20.

McCallum, Bennett T. "Some Issues Concerning Interest Rate Pegging, Price Level Determinacy, and the Real Bills Doctrine." *J. Monetary Econ.* 17 (January 1986): 135–60.

———. *Monetary Economics: Theory and Policy*. New York: Macmillan, 1989.

Marcet, Albert, and Sargent, Thomas J. "Convergence of Least Squares Learning Mechanisms in Self-referential Linear Stochastic Models." *J. Econ. Theory* 48 (August 1989): 337–68.

Patinkin, Don. "On the Short-Run Non-Neutrality of Money in the Quantity Theory." *Banca Nazionale del Lavoro Q. Rev.*, no. 100 (March 1972), pp. 3–22.

Poole, William. "Optimal Choice of Monetary Policy Instruments in a Simple Stochastic Macro Model." *Q.J.E.* 84 (May 1970): 197–216.

Rotemberg, Julio J. "A Monetary Equilibrium Model with Transactions Costs." *J.P.E.* 92 (February 1984): 40–58.

Sargent, Thomas J. *Macroeconomic Theory*. 2d ed. New York: Academic Press, 1987.

Sargent, Thomas J., and Wallace, Neil. " 'Rational' Expectations, the Optimal Monetary Instrument, and the Optimal Money Supply Rule." *J.P.E.* 83 (April 1975): 241–54.

Wicksell, Knut. *Lectures on Political Economy*. Vol. 2 *Money*. 1905. Translated by Ernest Classen and edited by Lionel C. Robbins. London: Routledge, 1935.

Woodford, Michael. "Learning to Believe in Sunspots." *Econometrica* 58 (March 1990): 277–307. (*a*)

———. "The Optimum Quantity of Money." In *Handbook of Monetary Economics*, edited by Benjamin Friedman and Frank H. Hahn. Amsterdam: North-Holland, 1990. (*b*)